Review of the Modified Berggren Equation for the Prediction of Frost Depth in Coarse-grained Soils and Design Air Freezing Indices

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ABSTRACT
Predicting the depth of frost penetration is an important consideration for many construction projects and the modified Berggren equation (Aldrich and Paynter, 1953) is still one of the most widely used methods. However, there is confusion in the literature surrounding the modified Berggren equation and supporting equations/inputs, which can make using this method difficult. This paper provides a review of the modified Berggren equation and the various input parameters required for predicting the depth of frost penetration in coarse-grained soils. A detailed review and assessment of some of the common methods for estimating design air freezing indices are also presented, including the approach that is provided in the Canadian Foundation Engineering Manual (2006). Based on the results of the assessment that was completed for 11 sites in the Canadian province of Newfoundland and Labrador, the approaches/assumptions related to design freezing indices that have been proposed in the literature could not be validated, due in part to the natural variability associated with historical weather data. The use of inaccurate design air freezing indices can result in overdesign and unnecessary costs to clients and owners, or it could lead to design deficiencies, resulting in expensive repair costs that might have otherwise been avoided. Using design indices that are based on statistically determined return periods is a good step that practitioners can take towards improving design capabilities when it comes to predicting the depth of frost penetration.

RÉSUMÉ
La prédiction de la profondeur de pénétration du gel est une considération importante pour de nombreux projets de construction. L'équation de Berggren modifiée (Aldrich et Paynter, 1953) est toujours l'une des méthodes les plus largement utilisées. Cependant, il existe une confusion dans la littérature concernant l'équation de Berggren modifiée et les équations / entrées de support, ce qui peut rendre l'utilisation de cette méthode difficile. Cet article présente une revue de l'équation de Berggren modifiée et les divers paramètres d'entrée nécessaires pour prédire la profondeur de pénétration du gel dans les sols à gros grains. Un examen et une évaluation détaillés de certaines des méthodes courantes d'estimation des indices de congélation de l'air de conception sont également présentés, y compris l'approche fournie dans le *Canadian Foundation Engineering Manual* (2006). Sur la base des résultats de l'évaluation qui a été réalisée pour 11 sites dans la provinces canadienne de Terre-Neuve-et-Labrador, les approches/hypothèses liées aux indices de gel de conception qui ont été proposées dans la littérature n'ont pas pu être validées, en partie à cause de la variabilité naturelle associé aux données météorologiques historiques. L'utilisation d'indices de congélation à l'air de conception inexacts peut entraîner une conception excessive et des coûts inutiles pour les clients et les propriétaires, où elle peut entraîner des défauts de conception, entraînant des coûts de réparation coûteux qui auraient autrement pu être évités. L'utilisation d'indices de conception basés sur des périodes de retour déterminées statistiquement est une bonne étape que les praticiens peuvent franchir pour améliorer les capacités de conception lorsqu'il s'agit de prédire la profondeur de pénétration du gel.

1 INTRODUCTION

The modified Berggren equation was developed by Drs. H.P. Aldrich and H.M. Paynter in 1953 and is presented in technical guidelines in both the United States and Canada as an acceptable method for completing frost depth predictions. However, there is confusion in the literature related to the modified Berggren equation. The author has read technical guidelines and reports, including the Canadian Foundation Engineering Manual (CFEM) 4 ed. (2006), that contain discrepancies making the application of the equation difficult. In many cases, this results in a practitioner relying on a nomograph to obtain an estimate of the depth of frost penetration.

The practitioner using the nomograph can easily obtain a frost depth prediction but may not understand how the nomograph was developed or the limitations of its use. In addition, many of the available nomographs have an upper limit to the freezing index and this requires the user to extrapolate beyond the limits of the nomograph, particularly for northern climates in Canada and the United States. Use of the modified Berggren equation would eliminate these drawbacks and, when used correctly, should allow for a better estimate of frost penetration depth. This paper presents the pertinent details of the modified Berggren equation to aid practitioners when making estimates of the depth of frost penetration in coarse-grained soils that have a low potential to develop significant ice lenses. The application of the equation to highly frost susceptible soils, fine-grained soils, and/or stratified soil deposits requires some additional considerations that are not discussed in this paper. The information presented herein will allow the practitioner to work in either the Imperial or SI systems of measurement.

Many of the technical guidelines that reference the modified Berggren equation also include a recommendation that a design freezing index should be used to estimate the depth of frost penetration. For design indices, common practice is to use the highest air freezing index within a particular period, e.g., the highest air freezing index within the last 10 years, or the average of the three highest air freezing indices within the last 30 years. In addition, empirical relationships (e.g., Horne, 1987), which is presented in the CFEM (2006), have also been used to estimate a design freezing index.

In this paper, the results of an assessment that was completed to evaluate the different approaches for determining the design freezing indices mentioned above are presented. To calculate the various design indices, daily weather (temperature) records were downloaded from the Environment Canada historical weather online database, as well as normals data from the Environment Canada climate normals online database. The data was downloaded for 11 sites within the Canadian province of Newfoundland and Labrador. The general location of each site can be seen in Figure 1.

To conclude the design indices assessment, all available daily weather records for 4 of the 11 sites (St. John's, Gander, Wabush, and Nain) were downloaded from the Environment Canada historical weather database. The air freezing index for each available year within the database was calculated and using statistical analysis, the air freezing indices associated with various return periods were determined. The previously calculated design freezing indices were then evaluated to assess the corresponding return period.

Figure 1: Overview of Newfoundland & Labrador, Canada, showing the general location of the 11 study sites.

2 MODIFIED BERGGREN EQUATION

The modified Berggren equation for a homogeneous, non-stratified soil is presented below. The symbol notation used herein is provided in Appendix A.

$$X = \lambda \sqrt{\frac{2k_f I_s}{L_s}} \quad [1]$$

It is important to note that the surface freezing index (I_s) must be expressed in degree-hours for the temperature scale of choice (°F or °C) if the above form of the equation is to be used. If the surface freezing index is expressed in degree-days, then Equation 2 below is the valid form.

$$X = \lambda \sqrt{\frac{48k_f I_s}{L_s}} \quad [2]$$

In the next section, details will be provided regarding the application of the modified Berggren equation, as well as additional discussion on the relevant parameters.

3 APPLICATION OF THE MODIFIED BERGGREN EQUATION

The reports by Aldrich & Paynter (1953) and Aldrich (1956), provide good discussion on each parameter that must be determined for use with the modified Berggren equation. A summary of these parameters is provided in the sections below. More detailed explanations have been provided in Appendix A.

3.1 Thermal Conductivity

In Equations 1 and 2, the thermal conductivity of frozen soil (k_f) is used. However, part of the work completed by Aldrich & Paynter (1953) looked at using the average thermal conductivity, $k_{avg} = \frac{k_f + k_u}{2}$, in their equation, rather than (k_f). The results of their research showed that using the average thermal conductivity produced better results; thus, the average thermal conductivity was recommended by Aldrich and Paynter (1953) for use in the modified Berggren equation (see below).

$$X = \lambda \sqrt{\frac{2k_{avg} I_s}{L_s}} \quad [3]$$

$$X = \lambda \sqrt{\frac{48k_{avg} I_s}{L_s}} \quad [4]$$

Ideally, laboratory testing would be completed on soil samples from the location of interest to determine the representative thermal conductivity values. In the absence of laboratory test data specific to the site, the work of

Kersten (1949) can be used to provide a preliminary indication of thermal conductivity.

The graphs prepared by Kersten (1949) for unfrozen and frozen soils were developed using the Imperial system of measurement and the thermal conductivity values are in Btu/hr in °F. For use in either Equation 3 or Equation 4, it is preferable to use Btu/hr ft °F. This requires converting inches to feet. Sanger (1963) has presented Kersten's original graphs in units of Btu/hr ft °F and copies of the modified graphs (Chart B-1 & Chart B-2), for coarse-grained soils have been presented in Appendix B.

The graphs prepared by Kersten (1949) have also been presented in the SI system of measurement by Johnston et al. (1981). Copies of those graphs (Chart B-3 & Chart B-4), for coarse-grained soils, have also been included in Appendix B. It should be noted that the units for thermal conductivity on those graphs is W/m K. The magnitude of a kelvin and a degree Celsius is the same, therefore W/m K can be expressed as W/m °C. For use in either Equation 3 or Equation 4 it is preferable to use kJ/hr m °C. This requires converting Watts (W) to kJ/hr using the conservation factor 1 W = 3.6 kJ/hr

Kersten (1949) has also provided the empirical equations that were used to develop the graphs. Values of $\mathrm{k_u}$ and $\mathrm{k_f}$ for sandy soils (< 50% silt and clay) with moisture contents not less than 1% can be found using the following equations:

$$\mathrm{k_u} = [0.7 \log \mathrm{w_{\%}} + 0.4]10^{0.01\rho_{\mathrm{d_u}}} \quad [5]$$

$$\mathrm{k_f} = 0.076(10)^{0.013\rho_{\mathrm{d_u}}} + 0.032(10)^{0.0146\rho_{\mathrm{d_u}}}\,\mathrm{w_{\%}} \quad [6]$$

Values of $\mathrm{k_u}$ and $\mathrm{k_f}$ for silt and clay soils (≥ 50% silt and clay) with moisture contents not less than 7% can be found using the following equations:

$$\mathrm{k_u} = [0.9 \log \mathrm{w_{\%}} - 0.2]10^{0.01\rho_{\mathrm{d_u}}} \quad [7]$$

$$\mathrm{k_f} = 0.01(10)^{0.022\rho_{\mathrm{d_u}}} + 0.085(10)^{0.008\rho_{\mathrm{d_u}}}\,\mathrm{w_{\%}} \quad [8]$$

The units for thermal conductivity in the four equations are Btu/hr in °F; therefore, the answers should be multiplied by 0.0833 to obtain the recommended units of Btu/hr ft °F.

A similar set of equations in SI units is not believed to exist; however, conversion to kJ/hr m °C can be completed using the following conversion factors.

1 Btu/hr ft °F = 1.73073 W/m °C

1 W/m °C = 3.6 kJ/hr m °C

In summary, Kersten (1949) reports that the equations for sandy soils were largely based on tests on sands that can be considered fairly clean. For sandy soils with a relatively high silt and clay content (e.g., 40%), conductivity values intermediate to those obtained using the coarse-grained and fine-grained equations might provide a more reasonable estimate. Aldrich (1956) reports that the thermal conductivity of clean, well-graded granular base course materials lies approximately midway between the values for sandy soils and silt and clay soils given by Kersten (1949).

3.2 Volumetric Latent Heat of Solidification

The latent heat of solidification expresses the change in thermal energy in a unit volume of soil when the soil moisture freezes without a change in temperature, i.e., at a constant temperature. The latent heat depends only on the amount of water in a unit volume of soil. The basic equation for the volumetric latent heat of soil moisture, where heat is being released (exothermic reaction) as water turns to ice, is given below:

$$\mathrm{L_s} = -\Delta\mathrm{H_{solid}}\,\mathrm{w}\,\rho_{\mathrm{d_u}} \quad [9]$$

The preferred units for this term for use in either Equation 3 or Equation 4 are Btu/ft^3 or kJ/m^3.

It should be noted that other authors, e.g., Berggren, 1943; Aldrich & Paynter, 1953; and Aldrich, 1956, have previous defined the latent heat that is released as the latent heat of fusion; however, the latent heat of fusion requires positive enthalpy and the absorption of heat. The phase change that we are concerned with is from a liquid to solid, not vice versa, and the change in enthalpy is negative. The latent heat term ($\mathrm{L_s}$) in the denominator of the modified Berggren equation is negative and the thermal conductivity term in the numerator has a negative temperature gradient in accordance with the second law of thermodynamics, resulting in a positive term under the radical. In short, respecting the fundamental theory results in a mathematically correct equation.

3.3 Surface Freezing Index

The surface freezing index (I_s) has two components, i.e., the air freezing index or design freezing index (see Section 3.5) and the surface transfer coefficient. The units for the indices are either degree-hours or degree-days for the temperature scale of choice (°F or °C), depending on which form of the modified Berggren equation is preferred by the user; however, the use of degree-days is more convenient. The surface transfer coefficient is dimensionless.

3.3.1 Air Freezing Index

The most accurate way to determine the air freezing index is to plot a cumulative degree-day versus time curve. The period is usually a year, and this allows the number of degree-days to be obtained for the baseline temperature of interest. In this paper, the baseline or reference temperature used was 0 °C (32°F), since the focus is on coarse-grained soils. To calculate a degree-day, the mean daily temperature is subtracted from the baseline temperature. Plotting degree-day accumulations (positive and negative degree-days) versus time (day) for an entire year of interest will produce the required curve. Once the curve is plotted, the total number of degree-days between the highest point in autumn and the lowest point the following spring is the air freezing index for that particular year. Figure 2 below provides an example of a cumulative degree-day versus time curve.

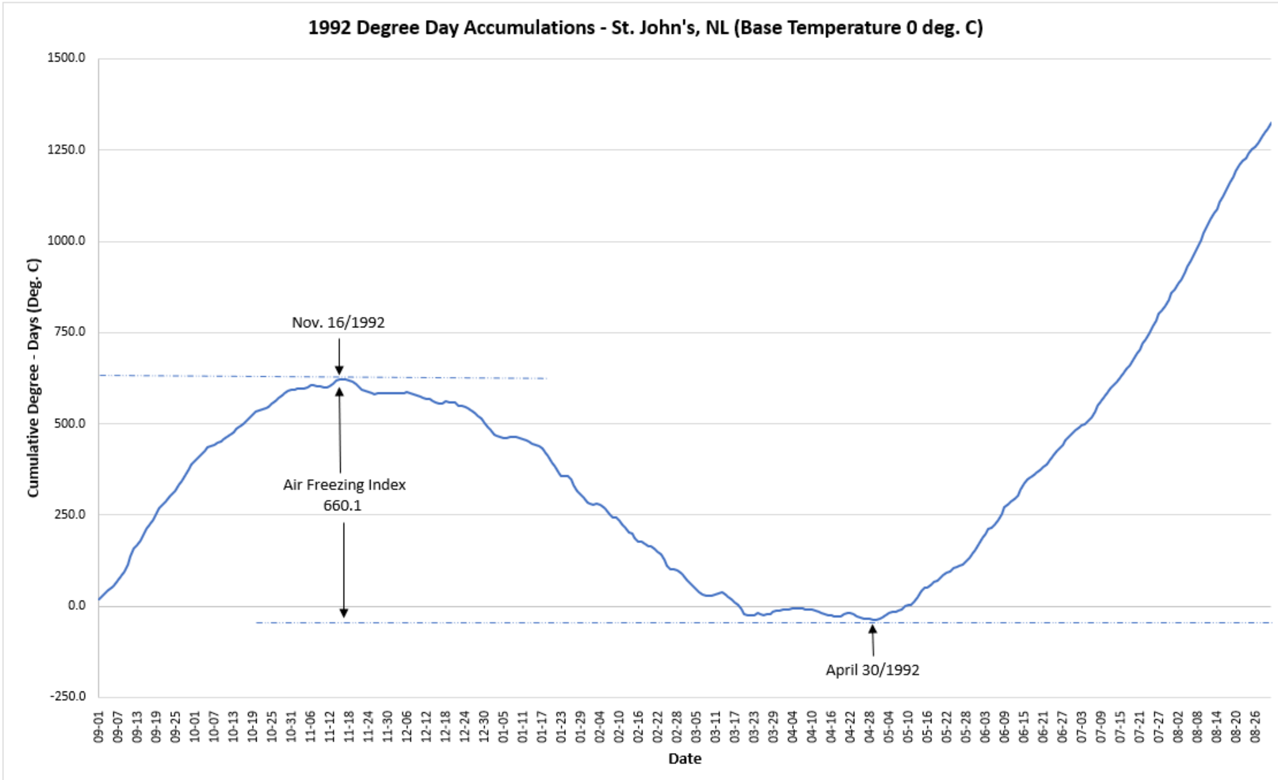

Figure 2: Cumulative °C-days versus time (1992-1993) curve; data obtained from Environment Canada weather station (Climate ID: 8403506) at St. John's, Newfoundland & Labrador, Canada.

Referencing Figure 2, the highest point in autumn was 623.2 °C-days and the lowest point in the following spring was negative 36.9 °C-days. The total number of degree-days between the two points is 660.1 °C-days. The 1992 air freezing index based on the mean daily temperatures is 660.1 °C-days and the length of the freezing period was from November 16 to April 30 (167 days). The change-over months were November and April.

3.3.2 Surface Transfer Coefficient

The depth of frost penetration is governed by the ground surface temperature; therefore, the surface transfer coefficient is a particularly important dimensionless parameter that is required to convert the air freezing index or design freezing index to the surface freezing index. The ground conditions can have a significant impact on the surface freezing index and in turn the predicted depth of frost penetration. Berggren (1943) concluded in his study that the depth of frost penetration for a given period with 4 inches (≅ 100 mm) of fresh snow cover on top of moist soil is only about one-sixth as great compared to the no snow cover condition.

Table 1 presents a summary of the various surface transfer coefficients that have been identified as part of a detailed literature review. In some of the reference documents, it is not clear if the surface transfer coefficients represent snow free or snow covered surfaces, and further detail has not been provided in Table 1 in those instances. Based on the data provided in Table 1, a summary table (Table 2) has been prepared to illustrate the range of values for different surface types and conditions. For Table 2, surfaces where a snow cover condition was not identified in Table 1, were assumed to have had snow removed.

Since most of the surface coefficients in Table 1 were obtained from studies completed in Alaska, it can be proposed that the values provided in Table 2 are generally applicable to sub-arctic climates within the temperate zone. For warmer climates within the temperate zone, the values on the lower end of the range might be more appropriate, or the data set provided in Table 1 could be updated to take into consideration the additional surface transfer coefficients that have been reported for concrete and asphalt (bituminous) pavements by Lunardini (1978). For instance, Lunardini (1978) provides additional values for states such as West Virginia, Iowa, Massachusetts, Minnesota, and New Hampshire.

3.4 Correction Coefficient

The correction coefficient (λ) is a function of three dimensionless parameters, i.e., the fusion parameter (μ), the root diffusivity ratio (δ), and the thermal ratio (α) (see the definitions in Appendix A for a detailed description of each parameter). The correction coefficient is used to correct the calculated depth of frost penetration in the modified Berggren equation and accounts for the volumetric heat capacity of both the frozen and unfrozen soil. The value of the correction coefficient can be obtained using Chart B-5 provided in Appendix B; the chart is based on a root diffusivity ratio of 1.0.

Table 1: Surface transfer coefficient data.

Surface Type	Transfer Coefficient (n)	Location/Comments	Reference
Spruce trees, brush, and moss over peat soil (snow cover present)	0.28	Fairbanks, Alaska	USACE, 1950a
Cleared of trees and brush with moss in place over peat soil (snow cover present)	0.25	Fairbanks, Alaska	USACE, 1950a
Silt loam cleared and stripped of trees and vegetation (snow cover present)	0.33	Fairbanks, Alaska	USACE, 1950a
Gravel (snow removed)	0.76	Fairbanks, Alaska	USACE, 1950a
Gravel (snow removed)	0.63	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete (snow removed)	0.74	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete (snow removed)	0.75	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete (snow removed)	0.81	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete (snow removed)	0.85	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete (snow removed)	0.69	Fairbanks, Alaska	USACE, 1950a
Pavement, bituminous (snow removed)	0.72	Fairbanks, Alaska	USACE, 1950a
Pavement, bituminous (snow removed)	0.78	Fairbanks, Alaska	USACE, 1950a
Pavement, bituminous (snow removed)	0.65	Fairbanks, Alaska	USACE, 1950a
Pavement, concrete	0.75	Bangor, Maine	USACE, 1950b
Pavement, concrete	0.78	Presque Isle, Maine	USACE, 1950b
Pavement, bituminous	0.87	Bangor, Maine	USACE, 1950b
Pavement, bituminous	0.95	Presque Isle, Maine	USACE, 1950b
Pavement, bituminous	0.92	Sioux Falls, S. Dakota	USACE, 1950b
Gravel	0.60	Alaska	Carlson & Kersten, 1953
Pavement, concrete	0.60	Alaska	Carlson & Kersten, 1953
Pavement, bituminous	0.60	Alaska	Carlson & Kersten, 1953
Pavement, unspecified (snow removed)	0.80	$I_a \leq 2{,}000\ (°F)$	McCormick, 1971
Pavement, unspecified (snow removed)	0.85	$2{,}000\ (°F) > I_a \leq 3{,}000\ (°F)$	McCormick, 1971
Pavement, unspecified (snow removed)	0.90	$I_a > 3{,}000\ (°F)$	McCormick, 1971
Crushed rock (snow cover)	0.7	Chitina, Alaska	Esch, 1973
Pavement, bituminous (snow removed)	1.0	Chitina, Alaska	Esch, 1973
Snow surface	1.0	Alaska & Greenland	JDAA, 1988
Bare soil (snow removed)	0.70	Alaska & Greenland	JDAA, 1988
Shaded surface (snow removed)	0.90	Alaska & Greenland	JDAA, 1988
Turf (snow removed)	0.50	Alaska & Greenland	JDAA, 1988
Pavement, concrete (snow removed)	0.75	Alaska & Greenland	JDAA, 1988
Pavement, bituminous (snow removed)	0.70	Alaska & Greenland	JDAA, 1988

Table 2: Summary of surface transfer coefficients provided in Table 1.

Surface Type	Transfer Coefficient (n)	Comments
Spruce trees, brush, and moss over peat soil (snow cover)	0.28	--
Cleared of trees and brush with moss in place over peat soil (snow cover)	0.25	--
Silt loam cleared and stripped of trees and vegetation snow cover)	0.33	--
Crushed rock (snow cover)	0.70	--
Turf (snow removed)	0.50	--
Bare soil (snow removed)	0.70	--
Gravel (snow removed)	0.60 – 0.76	Avg. = 0.66
Pavement, concrete (snow removed)	0.60 – 0.85	Avg. = 0.75
Pavement, bituminous (snow removed)	0.60 – 1.0	Avg. = 0.80
Pavement, unspecified (snow removed)	0.80	$\mathrm{I_a} \leq 2{,}000\ (°\mathrm{F} - \mathrm{days})$
Pavement, unspecified (snow removed)	0.85	$2{,}000\ (°\mathrm{F} - \mathrm{days}) > \mathrm{I_a} \leq 3{,}000\ (°\mathrm{F} - \mathrm{days})$
Pavement, unspecified (snow removed)	0.90	$\mathrm{I_a} > 3{,}000\ (°\mathrm{F} - \mathrm{days})$

The correction coefficient is always less than unity. For northern climates where the mean annual temperature approaches the freezing point of soil moisture, the thermal ratio approaches zero, and the correction coefficient is greater than 0.9 (Aldrich, 1956). If the mean annual temperature is below the freezing point of soil moisture, then the thermal ratio goes negative and this situation is not accounted for on Chart B-5 (Appendix B). Aldrich & Paynter (1953) indicate that, in such a case, a correction coefficient of 0.9 should be used.

3.5 Design Freezing Index

Until the early 1950s common practice was to use a mean air freezing index (I_m), based on a particular period of record (e.g., 30 years), to determine the surface freezing index for most design applications involving pavement and roadway construction (Linell, 1953). Linell (1953) proposed that a more significant freezing index, the design freezing index (I_d), should be used for design instead of a mean air freezing index. The initial suggestion was that the design freezing index could be based on an air freezing index that occurs 1-year-in-10, particularly in areas with a low mean air freezing index. The 1-year-in-10 air freezing index sounds like a return period; however, Linell (1953) indicates that a 1-year-in-10 air freezing index would correspond to the mean of the three highest air freezing indices in the latest 30 years of record, which is not an accurate method for determining a return period.

Linell et al. (1963) reports that the mean air freezing index of the three coldest winters in the latest 30 years of record is preferred for the design freezing index; however, if 30 years are not available, then the coldest winter in the latest 10-year period of record can be used. Like the previous definition (Linell, 1953), the assumption appears to be that the mean of the three coldest winters in the latest 30 years of record, can act as a proxy for the coldest winter in the last 10 years of the 30 years. This assumption is also presented in the CFEM (2006).

The JDAA (1987) technical manual for arctic and subarctic construction provides the same definition as Linell et al. (1963) for the design of permanent pavements. However, the definition of the design freezing index was expanded to include foundations. For the design of foundations for average permanent structures, the design freezing index should be based on the coldest winter in the latest 30 years of record or a corresponding frequency if continuous records are not available for the latest 30 years. It is important to note that the coldest winter in the latest 30 years of record, is not the equivalent of a 1:30 year return period.

Horne (1987) developed a relationship between the mean air freezing index and the design freezing index using climate data from 20 Canadian cities and proposed the following relationship.

$$I_d = 100 + 1.29\, I_m \qquad [10]$$

Horne (1987) reports that the design freezing index was taken as the coldest winter over the last 10-year period and, using that data and the mean air freezing indices from the 20 cities, the above relationship was developed. Table 3 is a reproduction of the data that Horne used to develop his equation.

Details are limited in Horne (1987) for his data set (Table 3). For instance, it is not entirely clear where the data was obtained from and how both air freezing indices (mean and design) were calculated. Horne (1987) mentions the climate normals (1931-1960) data that is presented in Boyd (1973) a couple of times throughout his thesis; however, after careful review of the data presented in Boyd (1973), it is unclear how the information in Boyd (1973) was used to obtain the mean air freezing indices presented in Table 3.

Table 3: Climatic data for 20 Canadian Cities (after Horne, 1987).

City	Air Freezing Index (°C-days)		Return Period (yrs.)
	Mean	Design	
Vancouver	45	236	44
Kamloops	463	798	30
Kelowna	463	638	12
Edmonton	1470	2173	44
Calgary	1168	1792	97
Saskatoon	1977	2669	39
Regina	1900	2889	87
Winnipeg	1920	2496	43
Thunder Bay	1532	1974	40
Sudbury	1418	1656	27
Ottawa	1058	1337	42
Toronto	638	834	42
Fredericton	896	1177	30
Charlottetown	736	1024	36
Shearwater	442	621	37
St. John’s	483	829	39
Niagara Falls	423	664	32
Quebec City	1191	1545	37
Montreal	958	1238	38
Trois Riveres	1164	1401	45

A review of the data presented in Table 3 helps illustrate an important point. In Table 3, the coldest winter in the last 10 year period is presented for each city; however, the return periods for each design freezing index are all more than 10 years and the highest return period was approximately 1:100-year. It is acknowledged that Horne does not assert that the design freezing index is equivalent to a 1:10-year return period, only the coldest winter over the last 10-year period. The point that must be made is that when selecting a design freezing index it is very important to understand what the index represents. For example, the design freezing index associated with a 1:10-year return period or 1:30-year return period will in all likelihood not be the same as the design freezing index based on the coldest winter in a continuous 10- or 30-year period.

The empirical relationship developed by Horne (1987) is presented in the CFEM (2006). In the manual, it is indicated that a long-term mean (30) year air freezing index can be estimated based on the monthly mean air temperature data published by Environment Canada (i.e., climate normals). It is worth noting that it is also mentioned in the CFEM (2006) that the use of Horne's relationship is recommended in the absence of an in-depth evaluation of historical weather data.

4 DESIGN FREEZING INDICES ANALYSIS – METHODOLOGY

The information contained within the CFEM (2006) is considered by many practitioners as the current state of practice. Given the general lack of information regarding how Horne's 1987 equation was developed and the lack of any additional recommendations in the CFEM (2006) on how to develop a design freezing index, an in-depth review of Horne's equation was completed to assess its applicability for determining the design freezing index based on climate normals data. In addition, analyses were also completed to determine if the mean of the three coldest winters in the 30 years of record for a particular climate normals period can act as a substitute for the coldest winter in the latest 10 years of the same period.

The most recent 30-year period for the Canadian climate normals that is available is 1981-2010. To complete the assessment, the climate normals data for the period 1981-2010 for 10 sites and the normals data for the period 1971–2000 for one site, all within the Canadian province of Newfoundland and Labrador, were obtained. Additionally, the Environment Canada daily weather records for the two periods mentioned above were downloaded. The 11 sites are as follows: St. John's, Harbour Breton, Gander, Stephenville, Main Brook, Plum Point, Cartwright, Goose Bay, Churchill Falls, Wabush, and Nain. The normals period 1971-2000 was used for the Churchill Falls site because more recent climate data are not available.

The climate and weather data were used to calculate design freezing indices for the 11 sites using three different approaches. The various design freezing indices were then compared and an evaluation of the different approaches was completed. The results of the data analysis and assessment of the design freezing indices are presented in the following sections.

4.1 Climate Normals Data (Approach 1)

The first approach that was used required an analysis of Environment Canada's climate normals database. The long-term mean air freezing indices for each site were based on the normals periods of interest, i.e., 1981 – 2010 and 1971 – 2000. Once calculated, the long-term mean air freezing indices were then used with Horne's equation to estimate the design indices for each site.

To calculate the long-term mean air freezing index, the change-over months needed to be identified and the associated number of air freezing degree-days determined. To estimate the number of air freezing degree-days for the change-over months, Boyd (1973) provides the following simplified equation:

$$\mathrm{D} = 0.5\mathrm{N}\left(\sqrt{\Delta \mathrm{v}^2 + 25} - \Delta \mathrm{v}\right) \qquad [11]$$

Where the temperature difference$(\Delta \mathrm{v})$ is defined as:

$$\Delta \mathrm{v} = \mathrm{v_a} - 32°\mathrm{F} \qquad [12]$$

N is the number of days in the month.

It is important to note that the equation yields °F-days. After reviewing Boyd, 1976, the above equation can be expressed as shown below to estimate the number of air freezing degree-days in °C-days.

$$\mathrm{D} = 0.5\mathrm{N}\left(\sqrt{\Delta \mathrm{v}^2 + 7.84} - \Delta \mathrm{v}\right) \qquad [13]$$

Where the temperature difference$(\Delta \mathrm{v})$ is defined as:

$$\Delta \mathrm{v} = \mathrm{v_a} - 0°\mathrm{C} \qquad [14]$$

To obtain a °C-days estimate using Equation 13, selection criteria for the change-over months must be defined. The change-over months can be generally defined as the last month near the end of winter where the mean daily temperature does not exceed 4.5°C and the first month where the mean daily temperature does not exceed 3.5°C near the start of winter.

It should be mentioned that this definition for the change-over months does not align with the definition provided in Appendix A. Also, while very similar, it does not follow exactly how Boyd (1973) accounted for the change-over months in his analysis. In addition, other definitions for the change-over months exist, i.e., McCormick (1991) defined a change-over month as a month where the monthly mean air temperature ranged between $\pm$ 2.5 °C.

For the months in between the change-over months, i.e., where the monthly mean air temperature is below 0°C, the number of below 0°C degree-days is found and the number of above 0°C degree-days for the same period is subtracted from the total. This value is then added to the number of degree-days determined for the change over months to obtain the cumulative air freezing degree-days for the site, i.e., the long-term mean air freezing index. Using Horne's equation, the design freezing index for the 11 sites was determined. For ease of reference throughout the remainder of this paper, these design indices will be referred to as "Horne's design indices".

Table 4 presents the long-term mean air freezing indices based on the climate normals and the design indices that were calculated using Horne's equation. In Table 4, the number of years with usable data within the normals period of interest is shown and the percentage of possible observations reported by Environment Canada is given. Both metrics indicate the completeness of each normals data set.

There are a couple of disadvantages to using a long-term mean air freezing index, based on the climate normals, to determine the design index, which includes:

1) To use the modified Berggren equation an estimate of the mean annual air temperature that corresponds to the design index is needed; however, a relationship has not been found to show how to determine the mean annual air temperature that corresponds to the design index.

2) For the modified Berggren equation, the duration of the freezing period is also required. Only a portion of the number of days in the change-over months contribute to the freezing period, and again no relationship was found during the literature review to show how to estimate an appropriate number of days for the change-over months.

Table 4: Climate normals data (mean annual air temperature and long-term mean air freezing indices) and design indices for 11 sites in NL, for the periods 1981 – 2010 and 1971 – 2000.

Site	Weather Station	Mean Annual Temperature Data			Long-term Mean Air Freezing Index Data			Horne's Design Indices
		°C	# Years	% Obs.	Long-term Mean Air Freezing Indices	# Years	% Obs.	
*St. John's	ID#8403506	5.0	30	100	441.9	30	100	670.1
Harbour Breton	ID#8402071	5.2	22	99.1	381.1	22	97.4	591.6
*Gander	ID#8401700	4.2	30	100	698.7	30	100	1001.3
*Stephenville	ID#8403800	5.0	30	99.7	589.4	29	99.5	860.3
Main Brook	ID#840KE88	2.0	22	100	1146.7	22	100	1579.2
Plum Point	ID#8402958	2.4	26	100	1032.1	25	99.7	1431.4
*Cartwright	ID#8501100	0.0	30	99.9	1514.3	28	98.3	2053.4
*Goose Bay	ID#8501900	0.0	30	100	1868.4	30	100	2510.2
Churchill Falls	ID#8501132	-3.7	23	100	2757.3	23	99.6	3656.9
*Wabush	ID#8504175	-3.1	27	99.9	2641.2	26	98.5	3507.1
Nain	ID#8502800	-2.5	27	99.9	2118.2	26	97.8	2832.5

* Station meets United Nations WMO standards. Freezing indices are in °C-days.

4.2 Daily Weather Data (Approach 2)

The second approach that was used to calculate the design air freezing indices for each site, required an in-depth analysis of the daily weather records available from the Environment Canada database for the normals period of interest for each of the 11 sites. The records were analyzed using a spreadsheet to calculate air freezing indices for each year. During the analysis, it was noticed that there were few weather stations with a continuous set of data available for each normals period, i.e., data was missing for an entire year(s) or daily temperature data was missing from a particular month(s) within a given year during the freezing season. Given the nature of the data, this is to be expected and, where possible, the data was used from a nearby weather station at the same site to fill in some of the data gaps. The maximum distance between weather stations with usable data was less than 1 km.

Using the daily weather records, air freezing indices were calculated for most of the sites and each year within the normal periods of interest. The results were then used to determine the three highest air freezing indices within the relevant normals period for each site so that the design indices (mean of the three highest air freezing indices) could be obtained. For ease of reference, these design indices will be referred to as the "3-year long-term mean design indices".

After careful review of the climate records, confidence is high that the three highest air freezing indices for each site for the normals period of interest were identified, despite some data gaps. Moreover, the three highest air freezing indices that were calculated for each site were from the same weather station and had almost continuous records for the relevant years. This allowed for a good comparison to be made with the design indices provided in Table 4, especially since both sets of indices are based on the data for the same normals period.

Table 5 summarizes the results obtained by plotting the cumulative degree-day versus time curves for the three highest air freezing indices for each site and presents the 3-year long-term mean design indices for each site. In addition, because individual cumulative degree-day versus time curves were constructed, an accurate value for the mean annual air temperature and the length of the freezing period for the design index can also be found. This information is provided in Table 5 and is required to use the modified Berggren equation. Preparing the cumulative degree-day versus time curves, also means that the degree-days associated with the change-over months are accurately accounted for.

Table 5: Summary of the 3-year long-term mean design indices for each site based on the normals period of interest, i.e., 1981-2010 and 1971-2000

Site	Weather Station	Year	Mean Annual Temp (°C)	Air Freezing Index (°C-days)	Length of Freezing Period (days)
St. John's	ID#8403506	1989	4.6	719.5	130
St. John's	ID#8403506	1992	3.6	660.1	166
St. John's	ID#8403506	1994	4.8	641.0	132
St. John's (3-year Long-term Mean Design Index)			4.3	673.5	143
Harbour Breton	ID#8402071	1989	4.6	562.2	129
Harbour Breton	ID#8402071	1990	4.7	541.0	99
Harbour Breton	ID#8402071	1992	4.0	542.2	163
Harbour Breton (3-year Long-term Mean Design Index)			4.4	548.5	130
Gander	ID#8401700	1989	3.9	1002.8	130
Gander	ID#8401700	1992	2.7	956.5	155
Gander	ID#8401700	1994	3.5	972.0	132
Gander (3-year Long-term Mean Design Index)			3.4	977.1	138
Stephenville	ID#8403800	1989	4.5	904.9	130
Stephenville	ID#8403800	1990	4.1	884.6	113
Stephenville	ID#8403800	1992	3.6	847.5	155
Stephenville (3-year Long-term Mean Design Index)			4.1	879.0	133
Main Brook	ID#840KE88	1990	1.0	1463.7	165
Main Brook	ID#840KE88	1991	0.3	1503.2	154
Main Brook	ID#840KE88	1992	0.5	1442.5	183
Main Brook (3-year Long-term Mean Design Index)			0.6	1469.8	167
Plum Point	ID#8402958	1990	1.0	1429.7	182
Plum Point	ID#8402958	1991	0.6	1436.2	153
Plum Point	ID#8402958	1992	0.8	1382.8	175
Plum Point (3-year Long-term Mean Design Index)			0.8	1416.2	170
Cartwright	ID#8501100	1984	-1.3	1966.8	185
Cartwright	ID#8501100	1991	-2.0	1931.8	184
Cartwright	ID#8501100	1992	-1.6	1958.3	189
Cartwright (3-year Long-term Mean Design Index)			-1.6	1952.3	186
Goose Bay	ID#8501900	1982	-1.8	2299.5	192
Goose Bay	ID#8501900	1984	-1.1	2246.1	187
Goose Bay	ID#8501900	1992	-1.8	2265.8	190
Goose Bay (3-year Long-term Mean Design Index)			-1.6	2270.5	190
Churchill Falls	ID#8501132	1972	-6.3	3557.8	218
Churchill Falls	ID#8501132	1974	-4.6	3114.9	208
Churchill Falls	ID#8501132	1992	-5.0	3101.2	206
Churchill Falls (3-year Long-term Mean Design Index)			-5.3	3258.0	211
Wabush	ID#8504175	1982	-4.4	2965.8	196
Wabush	ID#8504175	1989	-3.9	2978.7	196
Wabush	ID#8504175	1992	-4.8	3031.0	206
Wabush Mean (3-year Long-term Mean Design Index)			-4.4	2991.8	199
Nain	ID#8502800	1990	-4.4	2673.5	208
Nain	ID#8502800	1991	-4.6	2709.9	201
Nain	ID#8502800	1992	-4.7	2714.8	195
Nain Mean (3-year Long-term Mean Design Index)			-4.6	2699.4	201

4.3 Daily Weather Data (Approach 3)

For the third approach, the individual air freezing indices that were calculated for each site were used to identify the highest air freezing index based on the last 10 years of available data for the target periods 2001-2010 and 1991-2000, i.e., last 10 years within the normals period of interest. For both target periods, there were years where an accurate air freezing index could not be calculated for some sites; therefore, those years were excluded and additional years were included in the search so that 10 years of data were represented. This expanded the range of the target periods to 1997-2010 and 1989-2000 in some cases. The results are presented in Table 6 in the next section and a summary of the results is provided. For ease of reference throughout the remainder of this is paper these design indices will be referred to as the "10-year maximum design indices".

In summary, the climate normals data and the daily weather data were used to calculate three different sets of design indices, using three different approaches. Based on previous discussion, the design indices, either directly or indirectly, should represent a 10-year maximum design index, i.e., the highest air freezing index within the last 10 years for a particular normals period. A detailed evaluation and comparison of the design indices are presented in the next section.

5 DESIGN FREEZING INDICES ANALYSIS - RESULTS SUMMARY

Table 6 provides a comparison between Horne's design indices (column a) and the 3-year long-term mean design indices (column b) that were previously presented in Table 4 and Table 5, respectively. The difference in °C-days between the two sets of design indices (column c) and the percent error (column d) is also shown. For the percent error calculations, the 3-year long-term mean design indices can be considered the more accurate of the two sets of design indices, since they were obtained directly from an evaluation of the daily weather records. The results in Table 6 show that Horne's design indices overestimated the 3-year long-term mean design indices for 9 of the 11 sites. In terms of degree-days, the overestimation ranged from 15.2 °C-days to 515.3 °C-days. The percent error for all 11 sites ranged from 0.5% to 17.2% and ranged from 2.5% to 17.2% for the 9 sites that were overestimated.

A comparison between Horne's design indices (column a) and the 10-year maximum design indices (column e) is also presented in Table 6. For the percent error calculations, the 10-year maximum design indices can be considered the more accurate of the two sets of design indices, since they were obtained directly from an evaluation of the daily weather records. As previously mentioned, the period is greater than 10 years, because in some instances the most recent 10 years of records for the target periods (2001-2010 and 1991-2000) were unavailable. In those cases, the annual records before and closest to the target period were reviewed so that a corresponding amount of data (10 years) could be assessed. Table 6 shows that Horne's design indices overestimated the 10-year maximum design indices for all 11 sites. The overestimation in terms of degree-days (column f) ranged from 140.1°C-days to 611.3 °C-days. The percent error (column g) for all 11 sites ranged from 17.9% to 36.1%.

The two comparisons that have been completed so far have been aimed at evaluating the applicability of using Horne's equation to determine a design index that is equivalent to the mean of the three highest air freezing indices within the normals period, or the highest air freezing index for last 10 years of available data within the normals period.

As previously discussed, the mean of the three highest air freezing indices within the normal period and the highest air freezing index for the last 10 years within the normals period has been assumed by others to be roughly equivalent. To test the validity of this assumption a third comparison was completed.

The third comparison was completed from the point of view that the 3-year long-term mean design indices are the preferred indices for design. This was based on the recommendation by Linell et al. (1963) and the JDAA (1987), that the design index should be based on the mean of the three highest air freezing indices for a 30-year period and if that information is not available then the highest air freezing index in the last 10 years of the 30-year period could be used as a substitution. The results of the comparison between the 3-year long-term mean design indices (column b) and the 10-year maximum design indices (column e) are presented in Table 6. For the percent error calculations, the 3-year long-term mean design indices were used as the "true" value. The results in Table 6 show that the 10-year maximum design indices would underestimate the 3-year long-term mean design indices for all 11 sites if used as a substitution. The underestimate in terms of degree-days (column h) ranged from 96.0 °C-days to 364.8 °C-days. The percent error (column i) for all 11 sites ranged from 3.2% to 25.8%.

6 DESIGN FREEZING INDICES ANALYSIS – DISCUSSION

In the following discussion, a percent error of 10% or less has been selected as an acceptable error when comparing two sets of design indices. The rationale for selecting a percent error of 10% is based on previous analyses completed by the author, which have not been presented in this paper.

The results show that Horne's design indices agreed reasonably well with the 3-year long-term mean design indices for 8 of the 11 sites. The percent error for the 8 sites was less than 10% and the greatest difference in degree-days was 133.1°C-days. For the other three sites, the percent errors were 10.6%, 12.2% and 17.2% and the difference in degree-days were 239.7 °C days, 398.9° C-days and 515.3°C days, respectively.

Horne's equation predicted the design indices reasonably well for the 8 sites, where the long-term mean freezing index is approximately 2100 °C-days or less. Albeit the results for Goose Bay (long-term mean freezing index = 1868.4 °C-days; percent error = 10.6) was an anomalous result, as a smaller percent error would have been expected. The results for the two remaining sites with the higher percent errors suggest that Horne's equation is not a reliable predictor of the 3-year long-term mean design index when the long-term mean freezing index is roughly 2600 °C-days or higher.

Table 6: Comparison of the various design indices; design indices and degree-days reported in °C-days.

Site	Horne's Design Indices (Table 4) (a)	3-year Long-term Mean Design Indices (Table 5) (b)	Degree-Days Difference (a-b) (c)	% Error (d)	10-year Maximum Design Indices (e)	Degree-Days Difference (a-e) (f)	% Error (g)	Degree-Days Difference (e-b) (h)	% Error (i)
St. John's	670.1	673.5	-3.4	0.5	530.0 (2001-2010)	140.1	26.4	-143.5	21.3
Harbour Breton	591.6	548.5	43.1	7.9	446.9 (1997-2010)	144.7	32.4	-101.6	18.5
Gander	1001.3	977.1	24.2	2.5	765.9 (2001-2010)	235.4	30.7	-211.2	21.6
Stephenville	860.3	879.0	-18.7	2.1	692.2 (2001-2010)	168.1	24.3	-186.8	21.3
Main Brook	1579.2	1469.8	109.4	7.4	1230.0 (1997-2010)	349.2	28.4	-239.8	16.3
Plum Point	1431.4	1416.2	15.2	1.1	1051.4 (1998-2010)	380.0	36.1	-364.8	25.8
Cartwright	2053.4	1952.3	101.1	5.2	1667.5 (2001-2010)	385.9	23.1	-284.8	14.6
Goose Bay	2510.2	2270.5	239.7	10.6	2122.8 (2001-2010)	387.4	18.2	-147.7	6.5
Churchill Falls	3656.9	3258.0	398.9	12.2	3101.2 (1989-2000)	555.7	17.9	-156.8	4.8
Wabush	3507.1	2991.8	515.3	17.2	2895.8 (2001-2010)	611.3	21.1	-96.0	3.2
Nain	2832.5	2699.4	133.1	5.0	2347.7 (2001-2010)	484.8	20.6	-351.7	13.0

Note: Negative values for the difference between two design indices indicate an underestimate of the design index from the expected (''true") value.

Comparing Horne's design indices with the 10-year maximum design indices, which is what Horne's equation is suppose to directly estimate (CFEM, 2006; Horne, 1987), shows that Horne's equation is inaccurate. The percent errors were all relativity high (17.9% - 36.1%) and using Horne's equation leads to an overestimation of the 10-year maximum design index for all 11 sites. Therefore, the ability of Horne's equation to provide a reasonable estimate of the coldest winter in the last 10 years of the normals period of interest, is not supported based on the results of the analyses completed.

Comparing the 3-year long-term mean design indices with the 10-year maximum design indices shows that the 10-year maximum design indices underestimate the 3-year long-term mean design indices for all 11 sites. The percent errors for most of the sites were relatively high, i.e., 13.0% to 25.8%, except for 3 sites in Labrador (Goose Bay, Churchill Falls, and Wabush) where the percent error was 6.5%, 4.8% and 3.2%, respectively. These results show that assuming the highest air freezing index for the last 10 years of available data within the normals period is equivalent to the mean of the three highest air freezing indices within the normals period and, conversely, is not valid for most of the sites.

Evaluation of the entire data set for the normals periods of interest revealed that Horne's design indices exceeded all individual air freezing indices that were calculated for 7 of the 11 sites, the exceptions were St. John's, Gander, Stephenville, and Plum Point. This finding shows that Horne's equation can lead to design indices that are higher than the maximum air freezing index for the entire period upon which the long-term mean air freezing indices that were used in the equation are based.

The inherent variability of weather data within a large data set (e.g., 30-year period) and the potential clustering of the three highest air freezing indices within the 30-year period, means that it can not be assumed that the highest air freezing index within the last 10 years of record within the normals period and the mean of the 3 highest air freezing indices within the normals period are necessarily equivalent to one another, and doing so can lead to the use of design indices that do not meet the original intent. Furthermore, Horne's equation can be unpredictable for determining design indices and can lead to highly variable results and inappropriate estimates of the design freezing index. The use of these approaches and assumptions to determine the design freezing index for a site should only be done as a last resort when a detailed review of the daily weather records is not possible. Even then, a judicious review of the design freezing index that is obtained must be completed based on experience and sound professional judgement to determine its applicability to a particular site.

Equally concerning, is that the approaches discussed in this paper for determining design indices do not take into consideration occurrence probability (i.e., return period) and, in turn, do not facilitate examination of exceedance risk. This disconnect promotes the imprudent application of design freezing indices which can lead to overdesign and unnecessary costs. It could also result in underdesign and the serviceability limits of the infrastructure could be exceeded, resulting in early repairs and/or replacement; again, costs that might have otherwise been avoided.

To gain a better understanding of the occurrence probability associated with the calculated design freezing indices, 4 of the 11 sites were selected and all available historical daily weather records contained within the Environment Canada online database were processed. Individual air freezing indices were then calculated for the entire period of record for each site and statistically analyzed. The objective of the analysis was to interpret past air freezing indices in terms of future probabilities of occurrence to allow comparison with the different design freezing indices that were calculated for the 4 sites. The data was compared to several theoretical probability distributions and was found to be adequately represented by a Normal distribution, i.e., the null hypothesis was not rejected at a significance level of 5%; therefore, the Normal distribution can be considered an accurate representation of the data with 95% confidence. The results of the analyses are presented in Table 7.

The information provided in Table 7 further illustrates that assuming an equivalency between Horne's design indices, the 3-year long-term mean design indices, and the 10-year maximum design indices is not justifiable. The range in return periods for the design freezing indices calculated for the 4 sites is considerable. Depending on the size and type of construction using a design index that has not been validated based on an in-depth review of historical weather data can have notable impacts on the cost of construction or may result in unnecessary repair costs that might have otherwise been avoided.

Table 7: Comparison of the various design indices against the equivalent return periods; design indices reported in °C-days.

Site	Horne's Design Index	Return Period (# data points)	3-year Long-term Mean Design Indices	Return Period (# data points)	10-year Maximum Design Indices	Return Period (# data points)
St. John's	670.1	1:30-year (76)	673.5	1:33-year (76)	530.0 (2001-2010)	1:4-year (76)
Gander	1001.3	1:30-year (80)	977.1	1:22-year (80)	765.9 (2001 -2010)	1:4-year (80)
Wabush	3507.1	1:125-year (55)	2991.8	1:6-year (55)	2895.8 (2001-2010)	1:4-year (55)
Nain	2832.5	1:50-year (48)	2699.4	1:20-year (48)	2347.7 (2001-2010)	1:4-year (48)

Note: # data points represent the number of individual years with usable data in the Environment Canada Database.

7 CONCLUSIONS

Based on the results of the assessment presented in this paper, the approaches/assumptions related to design freezing indices that have been proposed in the literature could not be validated, due in part to the natural variability associated with historical weather data.

The use of an inappropriate design freezing index coupled with an incomplete understanding of the modified Berggren equation can lead to a considerable overestimation or underestimation of the appropriate depth of frost penetration for a site. Erroneous frost depth predictions can result in unnecessary costs to clients and owners.

The additional costs are difficult to justify, considering that it is relatively easy to download the historical weather data from Environment Canada and complete a detailed assessment so that a design freezing index can be determined directly from the data for use at a particular site. The cost associated with completing a detailed assessment is minimal and can result in potential cost savings to clients and owners. Using design indices that are based on statistically determined return periods is a good step that practitioners can take towards improving design capabilities when it comes to predicting the depth of frost penetration.

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APPENDIX A

NOTATION

Symbol	Term/Description	Imperial Units	SI Units
α	Thermal ratio	Dimensionless	Dimensionless
a	Thermal diffusivity (frozen or unfrozen soil)	ft^2/hr	m^2/hr
a_u	Thermal diffusivity (unfrozen soil)	ft^2/hr	m^2/hr
a_f	Thermal diffusivity (frozen soil)	ft^2/hr	m^2/hr
A	Area	ft^2	m^2
c_s	Specific heat of soil minerals	Btu/lb °F	kJ/kg °C
c_w	Specific heat of water	Btu/lb °F	kJ/kg °C
c_i	Specific heat of ice	Btu/lb °F	kJ/kg °C
C	Volumetric heat capacity (frozen or unfrozen)	Btu/ft^3 °F	kJ/m^3 °C
C_u	Volumetric heat capacity (unfrozen soil)	Btu/ft^3 °F	kJ/m^3 °C
C_f	Volumetric heat capacity (frozen soil)	Btu/ft^3 °F	kJ/m^3 °C
C_{avg}	Average volumetric heat capacity (frozen and unfrozen soil)	Btu/ft^3 °F	kJ/m^3 °C
D	Degree-days	°F - day	°C - day
$-\Delta H_{solid}$	Enthalpy of solidification (water to ice)	Btu/lb	kJ/kg
δ	Root Diffusivity ratio	Dimensionless	Dimensionless
F	Freezing period (duration of freezing index)	Days	Days
G_s	Specific gravity of soil/mineral particles	Dimensionless	Dimensionless
i	Thermal gradient (negative)	°F/ft	°C/m
I_a	Air freezing Index	°F -days	°C -days
I_d	Design freezing Index	°F -days	°C -days
I_m	Mean air freezing Index	°F -days	°C -days
I_s	Surface freezing index	°F -days	°C -days
k	Thermal conductivity (frozen or unfrozen soil)	Btu/hr ft °F	kJ/hr m °C
k_u	Thermal conductivity (unfrozen soil)	Btu/hr ft °F	kJ/hr m °C
k_f	Thermal conductivity (frozen soil)	Btu/hr ft °F	kJ/hr m °C
k_{avg}	Average thermal conductivity (frozen and unfrozen soil)	Btu/hr ft °F	kJ/hr m °C
λ	Correction coefficient	Dimensionless	Dimensionless
L_s	Volumetric Latent heat of soil moisture	Btu/ft^3	kJ/m^3
μ	Fusion parameter	Dimensionless	Dimensionless
N	Number of days in a change-over month	Days	Days
n-factor	Surface transfer coefficient	Dimensionless	Dimensionless
Q	Rate of heat flow per hour	Btu/hr	kJ/hr
ρ_d	Dry density of frozen or unfrozen soil	lb_m/ft^3	kg/m^3
ρ_{d_u}	Dry (unfrozen) density of soil	lb_m/ft^3	kg/m^3
ρ_{d_f}	Dry (frozen) density of soil	lb_m/ft^3	kg/m^3
ρ_w	Density of water	lb_m/ft^3	kg/m^3
S	Degree of saturation	Decimal	Decimal
Δv	Temperature difference	°F	°C
v_a	Mean annual air temperature	°F	°C
$v_{\bar{o}}$	Degrees by which mean annual surface/air temperature exceeds the freezing point of soil moisture	°F	°C
$v_{\bar{s}}$	Degrees by which the mean subfreezing surface temperature is less than the freezing point of soil moisture during the freezing period.	°F	°C
w	Total gravimetric water content of soil	Decimal	Decimal
$w_{\%}$	Total gravimetric water content of soil	Percent	Percent
w_u	Gravimetric water content of unfrozen soil	Decimal	Decimal
w_i	Gravimetric ice content of soil	Decimal	Decimal
X	Depth of frost penetration	Feet	Metres

TYPICAL VALUES

Term	Symbol	Imperial Units	SI Units
Specific heat of soil minerals	c_s	0.17 Btu/lb °F	0.710 kJ/kg °C
Specific heat of water	c_w	1.00 Btu/lb °F	4.187 kJ/kg °C
Specific heat of ice	c_i	0.50 Btu/lb °F	2.094 kJ/kg °C
Enthalpy of solidification (water to ice)	$-\Delta H_{solid}$	-143.4 Btu/lb	-333.6 kJ/kg
Density of water	ρ_w	62.4 lb_m/ft^3	1000 kg/m^3
Specific gravity of soil/mineral particles	G_s	$2.6 < G_s \leq 2.8$	$2.6 < G_s \leq 2.8$

DEFINITIONS

A proper understanding of the terms below is required for the accurate application of the modified Berggren equation.

Air freezing index: The total number of degree-days between the highest point in autumn and the lowest point the following spring on a cumulative degree-day versus time curve for a freezing season. The index is a measure of the combined duration and magnitude of below freezing air temperatures during the freezing season. When determining the air freezing index, days where the mean temperature is above freezing cannot be ignored as there is a tendency for some ground to thaw on those days. The temperature departures above freezing must therefore be subtracted from the accumulations of temperatures below freezing. See also mean air freezing index and long-term mean air freezing index.

Change-over month: The autumn and spring months that include a seasonal maximum or minimum on the cumulative degree-day versus time curve.

Climate normals: The World Meteorological Organization (WMO) standard period for climate normals is 30 years. The normals should be arithmetic means of a weather variable (e.g., degree-days) for a given time of year (e.g., month). For normals values representing totals (e.g., degree-days), the WMO requires that an individual month be 100% complete for it to be included in the normals calculation.

Correction coefficient: Dimensionless correction coefficient used in the modified Berggren equation. The coefficient is a function of the fusion parameter, the diffusivity ratio, and the thermal ratio. The correction coefficient is used to correct the calculated depth of frost penetration in the modified Berggren equation and accounts for the volumetric heat capacity of the frozen and unfrozen soil.

Degree-day: The number of degrees in any one day that the mean daily temperature is above or below a given base temperature (e.g., 0 °C (32 °F)) is called a degree-day. For coarse-grained soils, soil moisture is assumed to freeze at 0 °C (32 °F). The difference between the daily mean temperature and the base temperature equals the degree-days for that day. A cumulative degree-day curve is obtained by plotting cumulative degree-day versus time for a certain period, usually for a specific year.

Design freezing index: A more significant air freezing index that is often used in design to account for a severe freezing season. Common practice is to choose the coldest winter within a reference period; however, the use of a return period is preferred.

Root Diffusivity Ratio: The square root of the diffusivity ratio. The diffusivity ratio measures the relative values of thermal diffusivity in the frozen and unfrozen soil. The root diffusivity ratio is expressed as:

$$\delta = \sqrt{\frac{a_f}{a_u}} = \sqrt{\frac{k_f C_u}{k_u C_f}}$$

Freezing period: The duration of the freezing index measured in days.

Fusion parameter: Measures the heat removed in the frozen soil (below the freezing point) compared to the latent heat of the soil moisture. The parameter can be initially defined as:

$$\mu = \frac{v_{\bar{s}} C_f}{L_s}$$

Where:

$$v_{\bar{s}} = \frac{I_s}{F}$$

Therefore:

$$\mu = \frac{C_f I_s}{L_s F}$$

As the amount of heat removed from the frozen soil (below the freezing point) increases, the thermal ratio generally tends to increase, therefore in colder climates, the fusion parameter would be greater than in warmer climates.

When $\mu = 0$, the only significant soil properties affecting the depth of frost penetration are the latent heat and the thermal conductivity of frozen soil. As the fusion parameter becomes large, the stored heat in the soil mass becomes proportionally more significant.

It can be assumed that the difference between the volumetric heat capacities C_u and C_f is not usually significant, therefore the C_f term can be replaced with the C_{avg} in the above equations and the fusion parameter can be redefined as:

$$\mu = \frac{v_{\bar{s}} C_{avg}}{L_s}$$

$$\mu = \frac{C_{avg} I_s}{L_s F}$$

Long-term mean air freezing index: Air freezing index based on monthly climate normals.

***Mean air freezing index*:** The arithmetic mean of three or more air freezing indices.

n-factor: Also known as the surface transfer coefficient. The n-factor is the ratio of the surface freezing index to the air freezing index. There is no simple correlation between the air freezing index and surface freezing index and an accurate determination of the coefficient for a specific location requires simultaneous measurements of air and surface freezing temperatures during a complete freezing season. Representative values of the n-factor are available in the literature, some of which will be presented in this report.

Specific heat capacity: The quantity of heat required to raise the temperature of a system by one degree. If the system is a unit mass of material (e.g., water, ice, soil) then it is commonly referred to as *specific heat capacity* or *heat capacity*.

Surface freezing index: A measure of the combined duration and magnitude of below freezing surface temperatures during the freezing season. The surface freezing index can be determined by multiplying the air freezing index by a surface transfer coefficient (n-factor) for a particular ground cover condition.

$$I_s = nI_a$$

For design, the surface freezing index is usually based on the design freezing index using the following equation.

$$I_s = nI_d$$

Thermal conductivity of soil: Expression of the rate of heat flow through a unit area under a thermal gradient. In other words, thermal conductivity is the quantity of heat flow in a unit time through a unit area of material (e.g., frozen or unfrozen soil) caused by a unit thermal gradient. The minus sign in the equation accounts for the negative temperature gradient, i.e., heat flows from a higher temperature to a lower temperature according to the second law of thermodynamics.

$$k = -\frac{Q}{i\,A}$$

Thermal diffusivity: Describes the rate of heat transfer through a material. The expression can be written as follows:

$$a = \frac{k}{\rho_d c_s} = \frac{k}{C}$$

Note: applicable subscripts u (unfrozen) and f (frozen) should be included in the above equation.

Thermal ratio: Measures the ratio of the heat stored initially in the unfrozen soil to the heat loss in the frozen soil. The ratio can be initially defined as:

$$\alpha = \frac{v_{\bar{0}}\, C_u}{v_{\bar{s}} C_f}$$

Where:

$$v_{\bar{s}} = \frac{I_s}{F}$$

It can be assumed that the difference between the volumetric heat capacities C_u and C_f is not usually significant, therefore the thermal ratio may be written as:

$$\alpha = \frac{v_{\bar{o}}}{v_{\bar{s}}}$$

From the above equation, the thermal ratio is the ratio of the degrees by which the mean annual surface temperature (it is assumed that the mean annual air temperature represents this parameter reasonably well) exceeds the freezing point of soil moisture, to the degrees by which the mean subfreezing surface temperature is less than the freezing point of soil moisture during the freezing period. The ratio can be expressed as follows:

$$\alpha = \frac{v_{\bar{o}}\, F}{I_s}$$

It is apparent that the thermal ratio decreases in colder climates and may be zero or negative.

Volumetric heat capacity of soil: Quantity of heat required to change the temperature of a unit volume of material (e.g., frozen or unfrozen soil) by one degree. The basic equation for volumetric heat capacity is given below:

$$C = \rho_d (c_s + c_w w_u + c_i w_i)$$

For mineral soil, unfrozen ($w_u = w, w_i = 0$)

$$C_u = \rho_{d_u} (c_s + c_w w_u)$$

For mineral soil, frozen ($w_i = w - w_u$); w_u assumed to be zero in completely frozen soil.

$$C_f = \rho_{d_f} (c_s + c_w w_u + c_i (w - w_u)$$

$$C_f = \rho_{d_f} (c_s + c_i w)$$

Where:

$$\rho_{d_f} = \frac{\rho_w}{\frac{1}{G_s} + (1.09 w_i + w_u)/S}$$

$$S = \frac{w}{\left[\frac{\rho_w}{\rho_{d_u}} - \frac{1}{G_s}\right]}$$

APPENDIX B

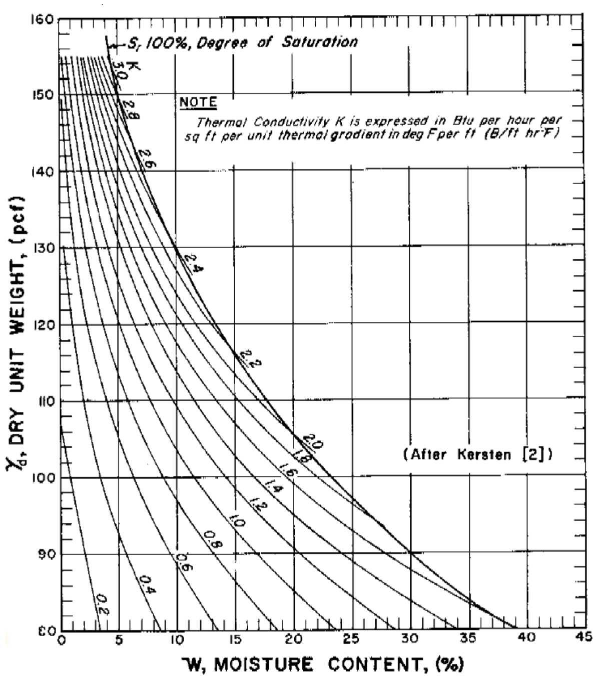

Chart B-1: Thermal conductivity of coarse-grained soil frozen (Sanger, 1966)

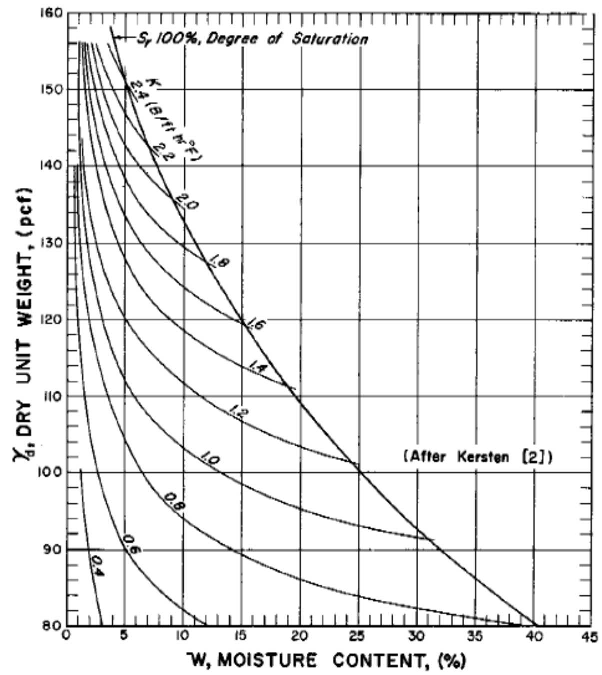

Chart B-2: Thermal conductivity of coarse-grained soil unfrozen (Sanger, 1966)

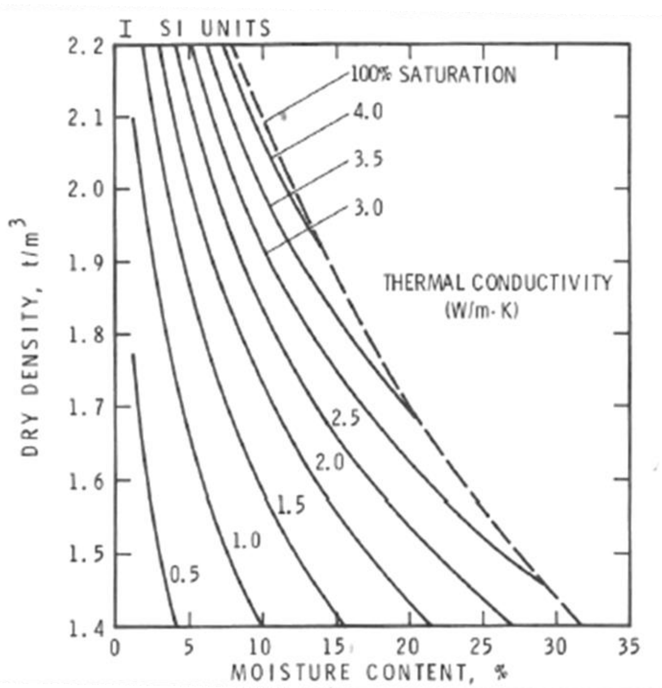

Chart B-3: Thermal conductivity of coarse-grained soil frozen (Johnston et. al. 1981)

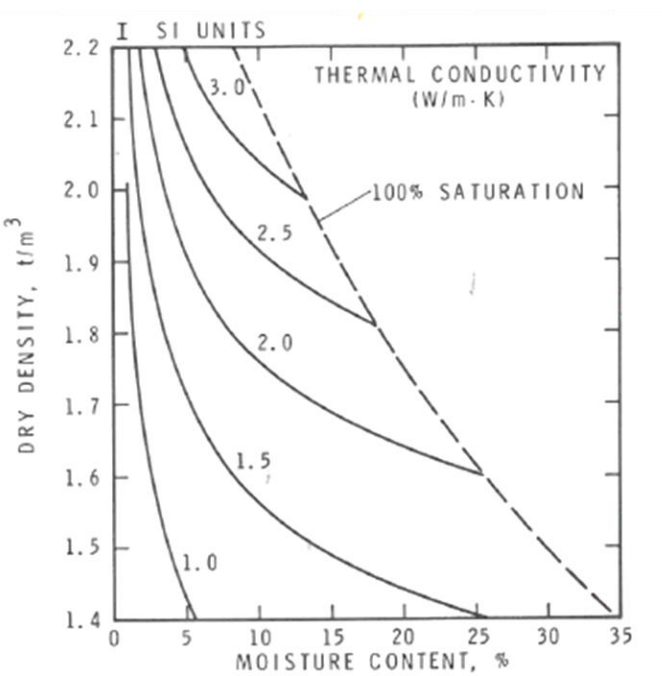

Chart B-4: Thermal conductivity of coarse-grained soil unfrozen (Johnston et al. 1981)

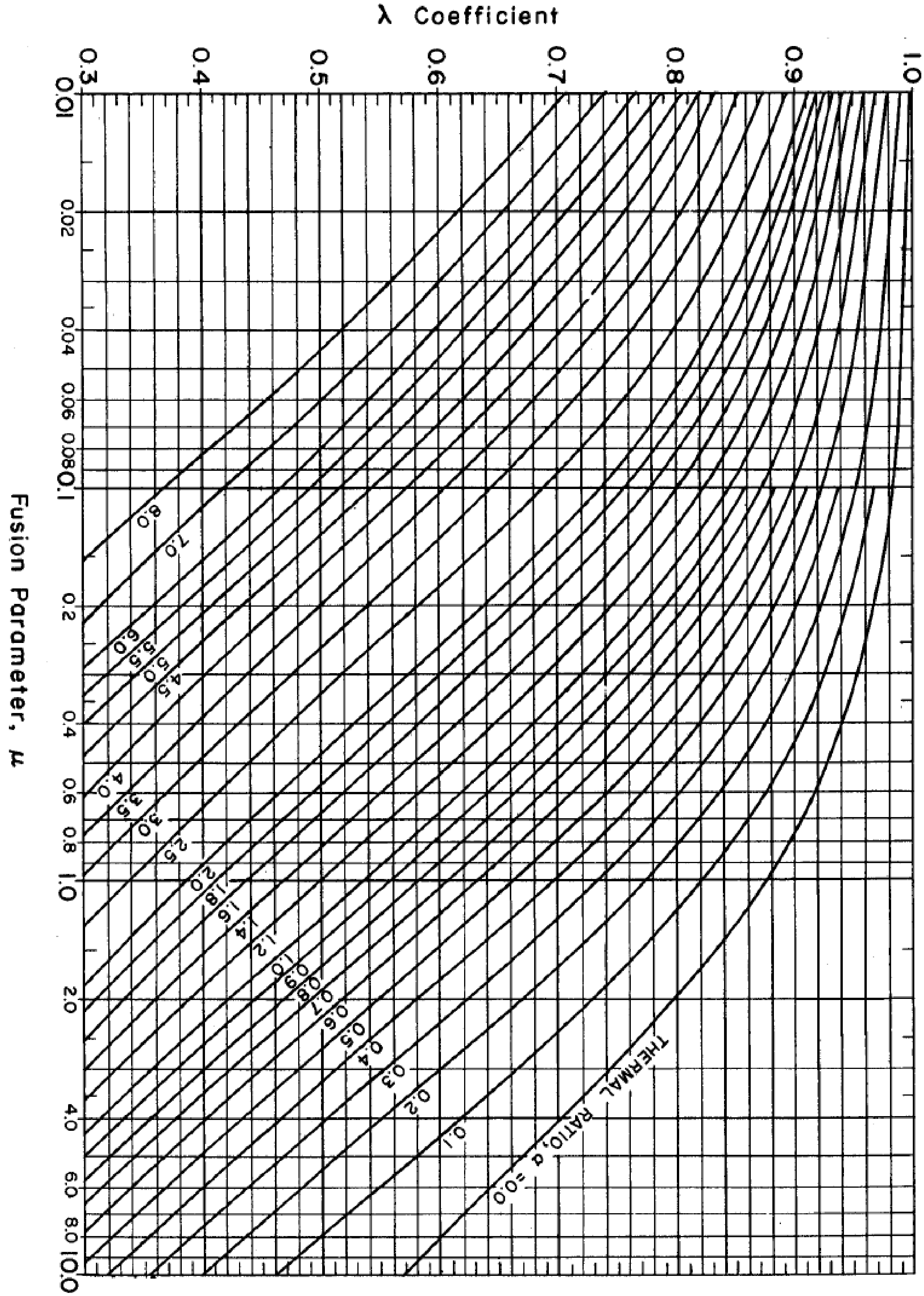

Chart B-5: Correction coefficient in the modified Berggren equation (modified from JDAA, 1988)