Internal Model vs Standard Formula

Modelling opportunities and challenges for market risks

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Agenda

1. Motivation and main insights from the EIOPA benchmark
2. Revisiting equity shocks with mean reverting models
3. A regime switching model for sovereign spread risk

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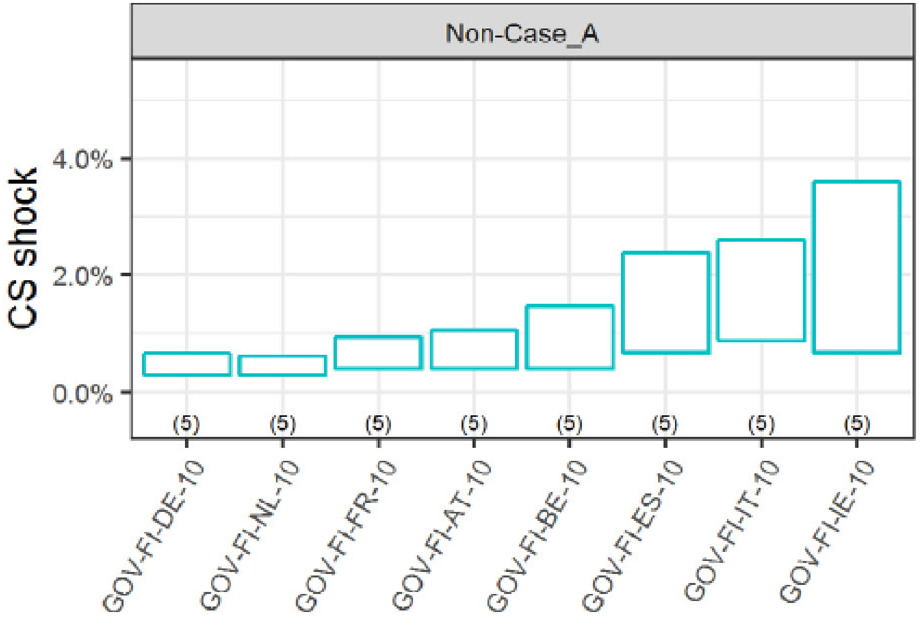
Motivation and main insights from the EIOPA benchmark

Motivation and main insights from the EIOPA benchmark

- While the sovereign spread risk is **not covered by the Standard Formula (SF)**, this risk is **often modelled within Internal Models (IMs)**
- In April 2021, EIOPA issued 2019 update of the annual Europe-wide **comparative study on the modelling of market and credit risk within IMs**, along with a comparison with the SF. As one of the main outputs from this benchmark study, it has been observed that **IM capital charges are materially driven by sovereign spread and default risks**.
- In particular, this study provides the distribution of the **1 in 200 annual spread shock** for 8 European countries (Germany, Netherlands, France, Austria, Belgium, Spain, Italy and Ireland):



- Note that these shocks are obtained from modular approaches modelling **only the credit spread risk**, in contrast to integrated approached modelling all facets of credit risk (default risk, migration risk, …) . In this presentation, we will also only focus on the credit spread risk.

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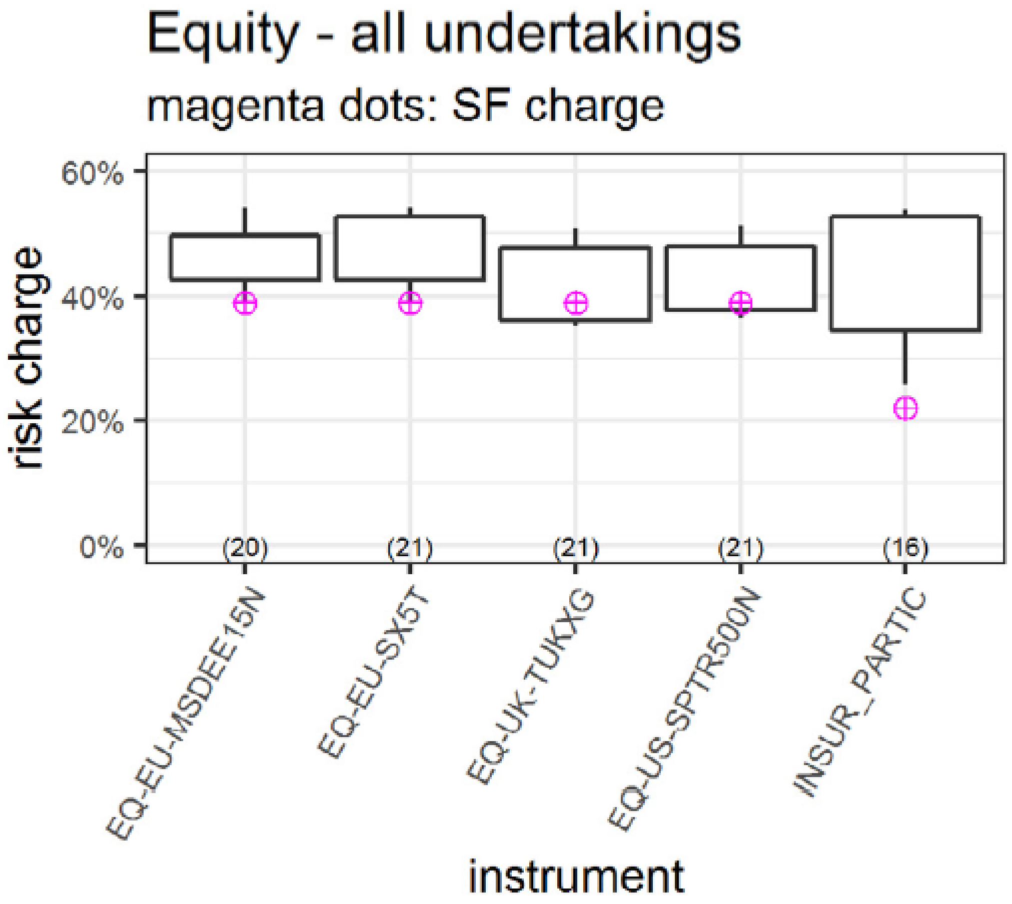
Source: EIOPA. Market and Credit Risk Comparative Study YE2019. https://www.eiopa.europa.eu/market-and-credit-risk-comparative-study-ye2019_en.

Motivation and main insights from the EIOPA benchmark

- With respect to **equity risk**, undertakings in general show less variation in the risk charges for major equity indices (as EuroStoxx 50, MSCI Europe, FTSE100 and S&P500) compared to risk charges applied to the strategic equity participation.
- Comparison with the SF on those main indices shows that the IM benchmark is disclosing **higher shocks**.
- This is to be balanced by observing that the risk charges applied by the undertakings with higher exposures tend to be closer to the SF than the average.
- For strategic equity investments (basically equity investments having a low volatility and for which the insurance undertaking has a clear strategy of holding its participation for a long period), IM benchmarks show **significantly higher shocks than the 22%** for such instruments.



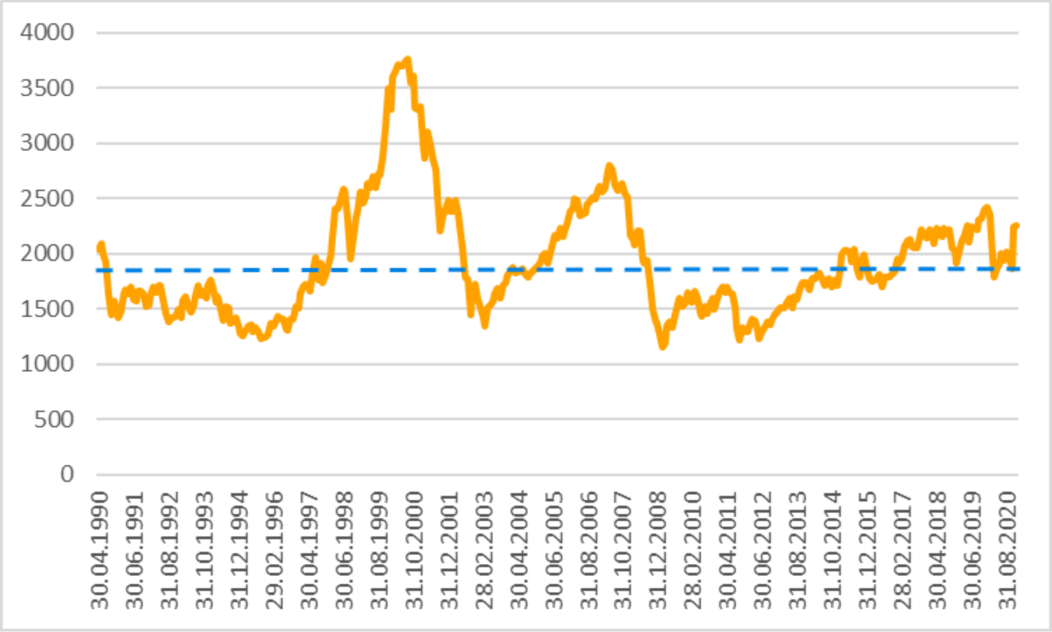
box contains sample results betw. 25%- and 75%-perc., whiskers extend to 10% and 90%-perc.; number in brackets is the overall sample size

For more insights, see https://fr.milliman.com/fr-fr/insight/EIOPAs-Market-and-Credit-Risk-Comparative-Study-Key-takeaways

Revisiting equity shocks with mean reverting models

Equity risk - Motivation

- The **equity risk sub-module** aims at **quantifying the impact of a sudden drop in the equity market** on the insurer balance sheet.
- At a refined time-step (intra-year in general), the stock market is known to be **very volatile** and has already notched up **significant crashes** in recent decades.
- However, after crisis, the **stock market tends to go back in average to its pre-crisis level** and even above after some time: this is referred to as the **mean-reverting behavior** of the stock market.
- These ideas are actually **not new**. Indeed, the Solvency II Directive already provides a specific treatment of three components, namely strategic equity investments, long-term equity investments and the duration-based approach.



Equity risk - A review of the Standard Formula shocks (1/2)

- The **equity shocks** are specified by the Solvency II Directive as follows:

$$Shock_{Type\ 1} = 39\% + SA$$

$$Shock_{Type\ 2} = 49\% + SA$$

where SA is the **symmetric adjustment**.

- The symmetric adjustment is an **adjustment factor** designed to prevent pro-cyclical effects of Solvency Capital Requirements (in particular, to avoid a rise in the equity risk charge in the middle of a crisis). It is defined by:

$$SA = \frac{1}{2}\left(\frac{CI - AI}{AI} - 8\%\right)$$

where:

- CI is the current level of an equity index representative of the equities held by the insurance undertaking,
- AI is the equally weighted average of the daily levels of the equity index over the last 36 months.

Equity risk - A review of the Standard Formula shocks (2/2)

- The standard shocks of **39%** and **49%** result from a calibration performed by EIOPA and detailed in a Calibration Paper published in 2010.
- The calibration of the shock for type 1 equities relies on daily data from the MSCI World Developed Index, spanning a period of 36 years (from 1973 to 2009).
- In three particular cases, a reduced shock can be applied instead of the standard approach. These three cases are:
  - Strategic equity investments (SEI)
  - Long-term equity investments (LTEI)
  - The "duration-based" (DB) approach

Category	Equity type	Shock	EIOPA calibration
Standard approach	Type 1	39% + SA	45% + SA
	Type 2	49% + SA	55% + SA
Strategic Equity Investments	All	22%	Ø
Long Term Equity Investment	All	22%	Ø
Duration Based approach	All	22%	22%

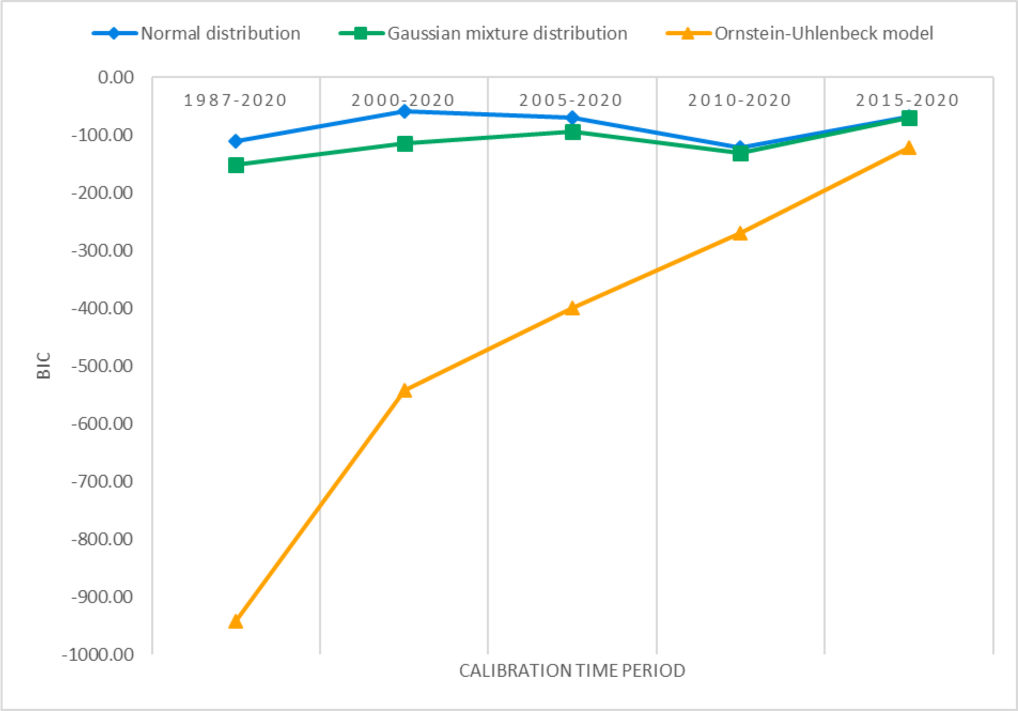
Equity risk – Calibration of mean reverting model

- We compute annual discounted log-returns using a rolling one-year window in line with the EIOPA approach and we consider the following set of models:

- **Normal distribution**
- **Gaussian mixture distribution**
- **Ornstein-Uhlenbeck model**, specified by the following stochastic differential equation:

$$dX_t = (\theta_1 - \theta_2 X_t)dt + \theta_3 dW_t$$

where X_t is the annual discounted log-return at time t and W_t is a Brownian motion.



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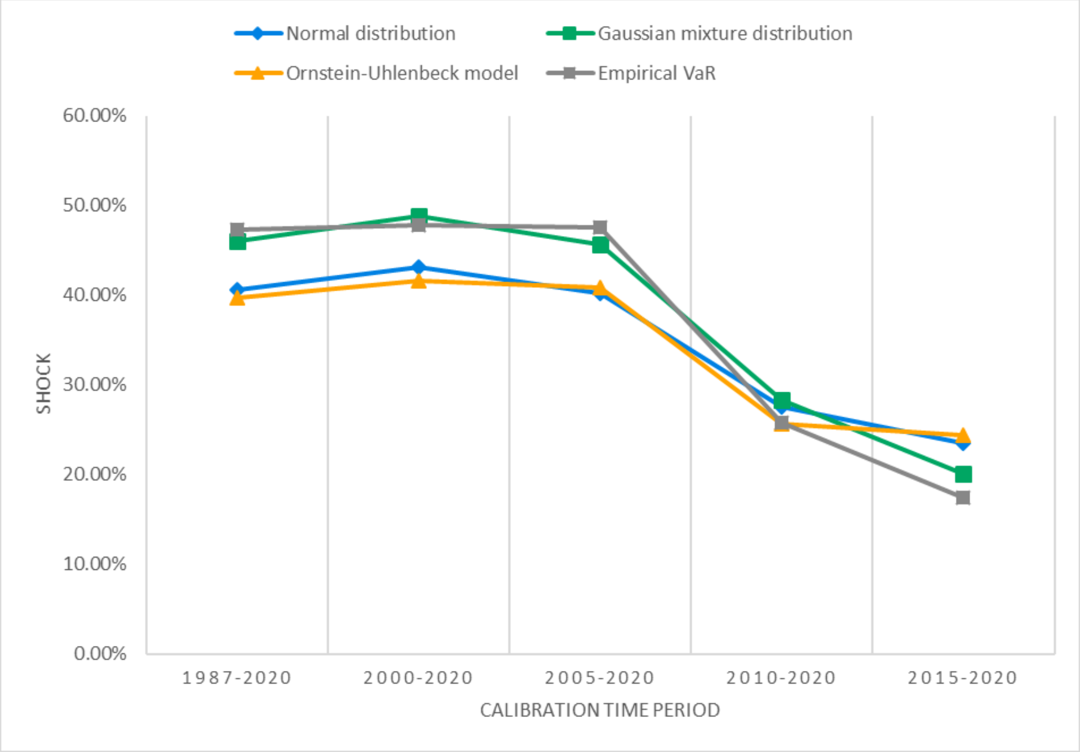
One year view vs long term view

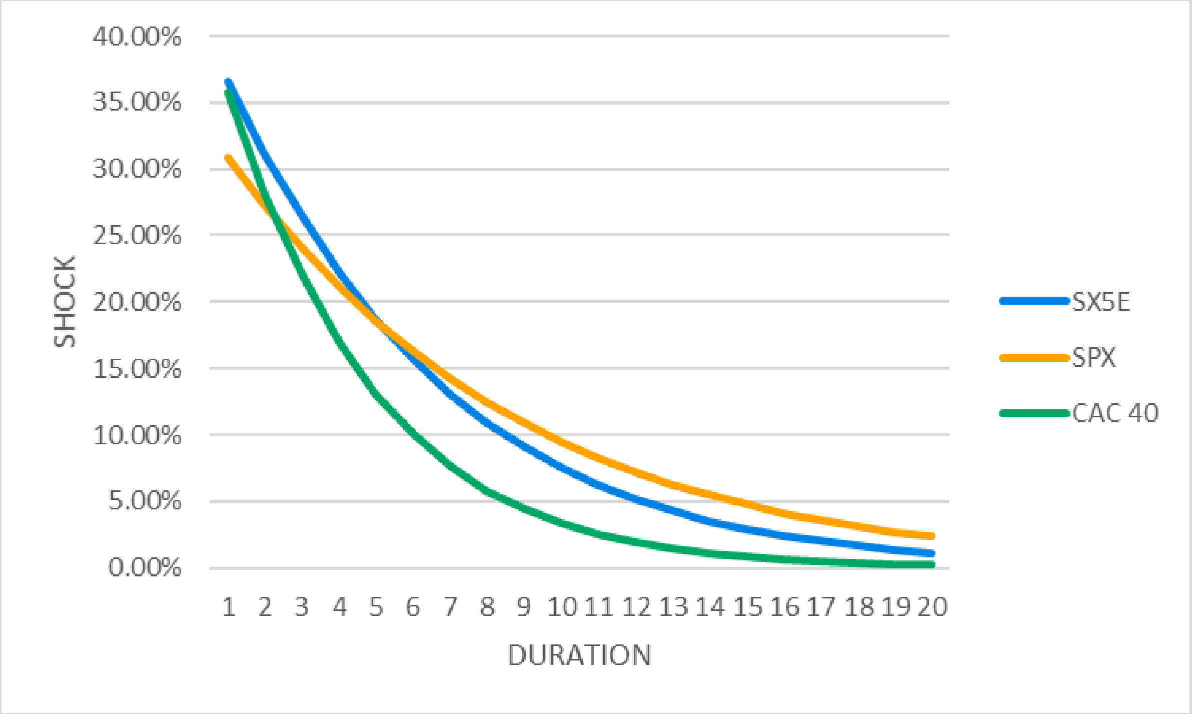
If we denote by S_t the value of the equity portfolio of an insurer at time t, then the **SCR of the equity sub-module** is given by:

$$SCR = VaR_{99.5\%}(S_0 - D_1 S_1) = S_0 - VaR_{0.5\%}(D_1 S_1)$$



We consider a duration of $T \geq 1$ years reflecting the time during which a synthetic asset is held, and we explore the following **alternative definition**:

$$SCR^* = \mathbb{E}[D_T S_T] - \mathbb{E}[D_T S_T | D_1 S_1 = VaR_{0.5\%}(D_1 S_1)]$$



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For more insights, see https://fr.milliman.com/fr-fr/insight/A-review-of-the-Solvency-II-equity-shock

A regime switching model for sovereign spread risk

Sovereign spread risk modelling - Motivation

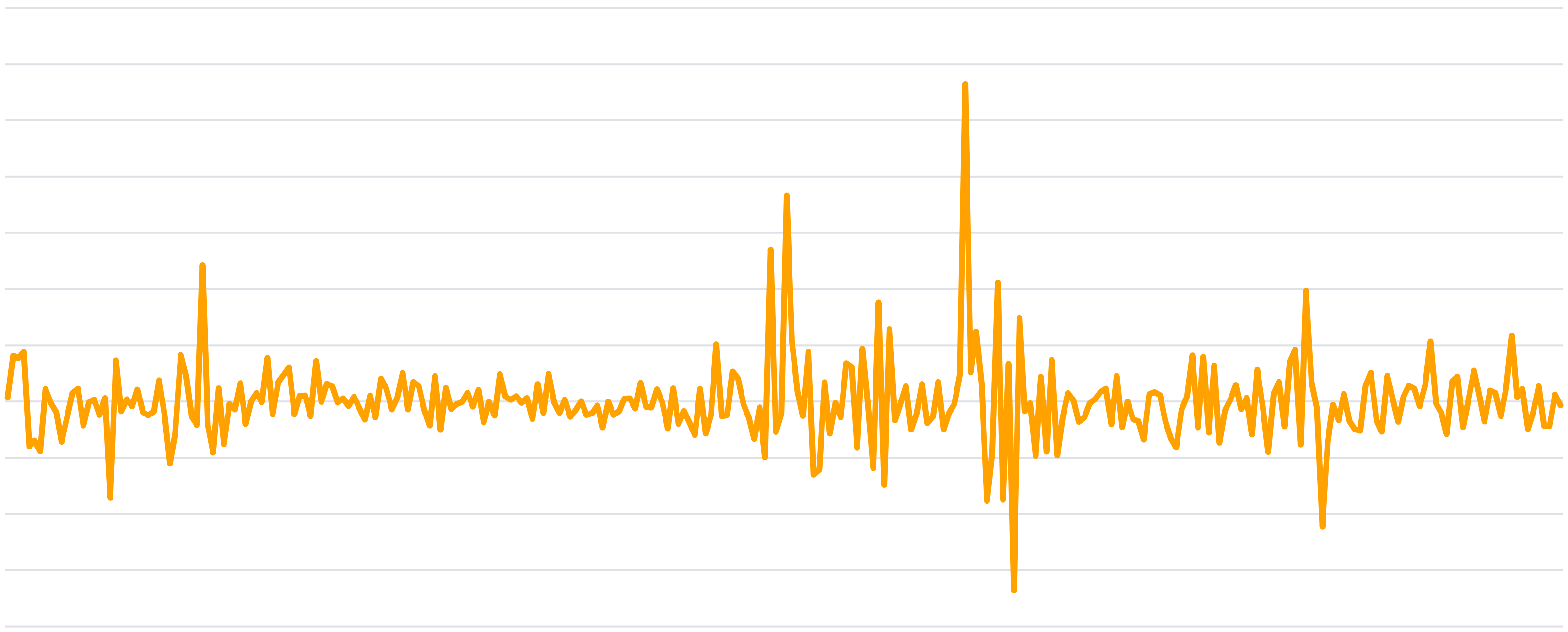
- In this section, we will present two modelling approaches **by country**, that is we consider a common model for all countries in portfolio but the model is calibrated specifically for each country.
- Such a modelling approach can be seen as **an improvement compared to a modelling by rating** (often used for the modelling of corporate credit risk) as in the latter, one assumes that countries that have the same rating also have the same spread risk profile and are highly correlated, which is not true in practice since the end of the European debt crisis in 2014.

- One can observe that the standard deviations can be quite **heterogeneous within the A and BBB rating groups** while they are quite homogeneous for the AAA and AA rating groups.

- Therefore, assuming the same level of risk for countries of the A or BBB rating groups could lead to **significant over- or underestimation of the capital requirements**.

Sovereign spread risk modelling - Data

- If we look at the historical time series of monthly spread increments for European countries since 1997, we can easily identify **two regimes of volatility**: **high volatility** during the subprime and the European debt crisis (from 2008 to 2014) and **low volatility** before that (from 2002 to 2008).



- This has motivated us to consider a **regime-switching (RS) model** with two regimes: a **high volatility regime** and a **low volatility regime**. The specification of the model is provided in the next slide.

Sovereign spread risk modelling - Model

- Let $(y_t)_t$ be a hidden **Markov chain with two states** (1 and 2) whose transition matrix is denoted by:

$$P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix}$$

  where p_{11} (resp. p_{22}) is the probability to stay in state 1 (resp. 2) given that the Markov chain is already in state 1 (resp. 2). The spread increments $X_t = s_t - s_{t-\delta}$ are then modelled as follows:

$$X_t = \begin{cases} \epsilon_t^1 & \text{if } y_t = 1 \\ \epsilon_t^2 & \text{if } y_t = 2 \end{cases}$$

  where $(\epsilon_t^1)_t$ and $(\epsilon_t^2)_t$ are i.i.d. normally distributed random variables with mean 0 and respectively variance σ_1^2 and σ_2^2.

- Note that the Markov chain $(y_t)_t$ **is not observed** and thus it is not an input that is used to calibrate the model. However, in order to calibrate the model one has to specify the initial state y_0 of the Markov chain. Several choices are possible for y_0 but in our numerical results we used a data-driven decision rule.

- Despite an additional layer of complexity, **the model remains quite parsimonious in terms of number of parameters** as there are only 4 parameters (σ_1, σ_2, p_{11}, p_{22}). Moreover, the model has **a clear economic justification and is supported by historical data**.

- An alternative distribution for the noises is the Student's t-distribution. However, we observed that the model calibration is less stable than with normally distributed noises.

Sovereign spread risk modelling – Calibration results

- In order to compare the different models, we rely on the **Bayesian Information Criteria** (BIC) which allows to measures the log-likelihood of a model with a penalization for the number of parameters and the number of data points, so that models with a lot of parameters having a high likelihood will not necessarily be the best according to the BIC. Hence, **the lower the BIC, the better the model**. Mathematically,

$$BIC = k \times \log(n) - 2 \times \text{LogLikelihood}.$$

where n is the number of data points.

- To calibrate the presented models, we proceed by log-likelihood maximization on historical monthly spread increments. We obtain the following values of BIC for 8 major European countries.

	Austria	Belgium	Czech Republic	France	Germany	Italy	Spain	Switzerland
Gaussian distribution	1947.1	2173.1	1614.7	1954.5	1868.8	2693.3	2572.3	1678.4
Student's t-distribution	1747.3	1934.8	1574.2	1830.0	1806.9	2477.2	2356.6	1549.1
Regime Switching model	1711.4	1905.2	1561.6	1807.3	1772.4	2353.2	2253.7	1518.5

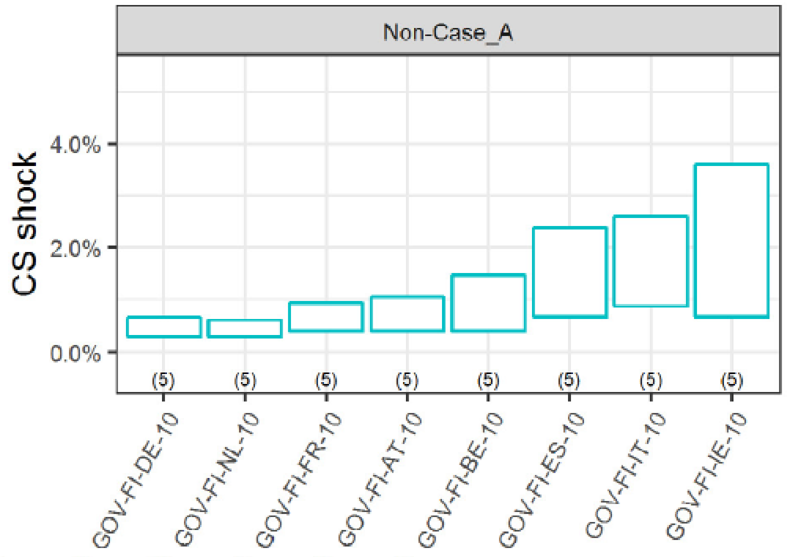
- It appears that the **RS models are clearly fitting better on historical data** than the single distribution models.

Sovereign spread risk modelling – Simulation results

- Once the models are calibrated, the quantity of interest for the insurer is the **99.5% Value-at-Risk (VaR)** of the one-year horizon spread shock. In both modelling frameworks, the annual spread shock is recovered by summing the modeled sub-annual spread shocks. For example, if we model monthly spread shocks X_t, the annual spread shock is given by:

$$S = \sum_{t=1}^{12} X_t$$

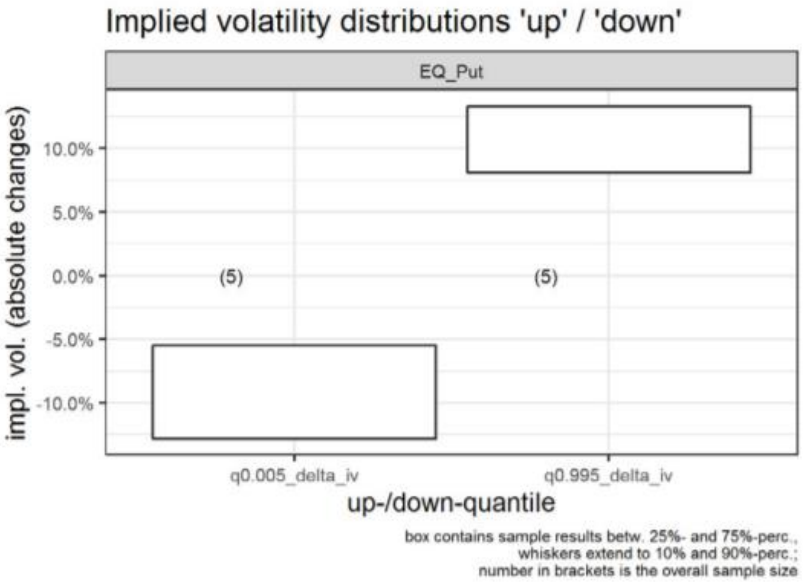
- Except in the case where the X_t's are i.i.d. and normally distributed, the distribution of S is unknown. Therefore, we rely on simulations to compute the 99.5% VaR.
- We observe that the **VaR's** obtained with the **regime-switching model are below the ones from a single student distribution**. Moreover, they **lie within the range of the EIOPA benchmark.**



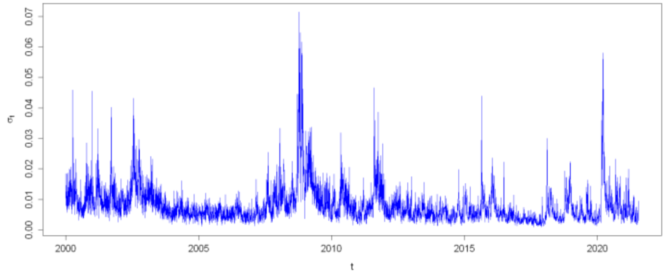
Going beyond

Going beyond

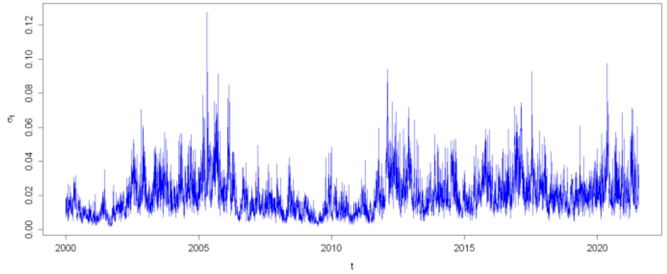
- The shocks from the benchmark are in a **relatively close range** for both **interest rate** and **equity-implied volatility shocks**.
- It is recalled that derivative positions on the asset side are not the only source of exposure to **implied volatility**, as this risk driver also **impacts the level of the value of options and guarantees on the liability side**.



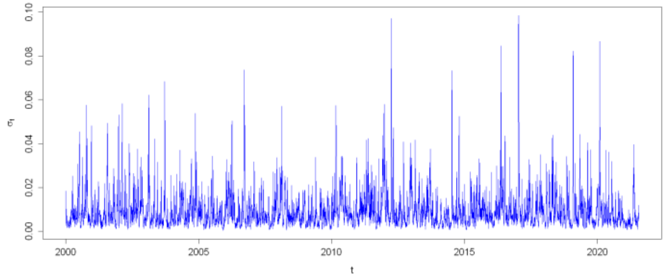
Equity volatility data



Fractional Brownian motion



Classical Brownian motion



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