ESG Rebase

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1. Introduction

Over the past 20 years, life insurers have been facing a sharply increasing demand for Economic Scenario Generator (ESG) calibrations:

- Only a handful of ESG sensitivity files were needed in the early days of stochastic modelling.
- From the mid-2000s, this increased following the introduction of the Realistic Balance Sheet and Individual Capital Adequacy Standards (ICAS) regimes in the UK.
- Solvency II produced a further marked increase with internal model firms, relying on traditional curve-fitting techniques, typically needing around 200 to 500 calibrations.
- Finally, internal model firms using Least Squares Monte Carlo techniques may use 25,000 or more ESG calibrations.

Further increases in demand for ESG calibration are to be expected whether they are in the International Financial Reporting Standard (IFRS) 17 context, for another regulatory purpose, for stochastic pricing, for business planning or for other purposes.

During this time, the models underlying a typical ESG have become more sophisticated, increasing the number of parameters involved and the dimensionality of the calibration problem.

Increased volume and sophistication have thus combined to make full ESG recalibrations increasingly onerous, highlighting the need for alternative approaches. The purpose of this paper is to introduce ESG Rebase—a technology capable of accurately translating a baseline risk-neutral valuation ESG file into another risk-neutral ESG file representing an entirely different set of market conditions, such as nominal yield curve, equity volatility vector or fixed interest volatility surface.

In a nutshell, ESG Rebase offers the following advantages:

- Required inputs are limited to a bare minimum
  - Base ESG file
  - Shock definitions
- Fully automated and scalable solution
  - No expert judgement needed
  - Even 1 million scenarios can be produced
- Full auditability via rich audit logs
  - Base file characteristics, e.g., volatilities
  - Target characteristics
  - Realised (rebased file) characteristics

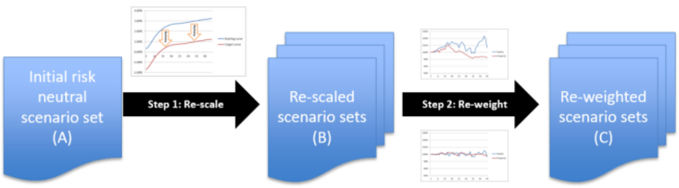
The aim of this paper is to describe and illustrate the methodology behind ESG Rebase from a position of practical experience. Milliman implemented the approach both as a desktop application in the Milliman STAR Solutions® - NAVI® software and as a component within our cloud-based Integrate® modelling solution.

This paper is organised as follows: In Section 2, we discuss the ESG Rebase algorithm and show how it combines classic rescaling techniques with much more advanced reweighting techniques known from banking. In Section 3, we demonstrate how this algorithm works on a few illustrative examples. In Section 4, we show how ESG Rebase serves as a backbone of an automated end-to-end proxy modelling process aligned to Solvency II internal model work. In Section 5, we conclude the article by considering further applications of ESG Rebase such as IFRS 17.

2. ESG Rebase algorithm

We can describe the ESG Rebase algorithm as a two-step process, illustrated in Figure 1.

FIGURE 1: ESG REBASE ALGORITHM



In the first step, a usual risk-neutral Base ESG file (A) is rescaled to implement the necessary changes to the initial nominal and real yield curves while maintaining the martingale property.

In the second step, the scenarios in the rescaled ESG file (B) are not changed as such anymore, but they are assigned certain non-uniform weights, so that the reweighted ESG file (C) would best satisfy volatility targets while also keeping an eye on the martingale property.

Now we are going to discuss these two steps in dedicated subsections.

2.1 RESCALING

As mentioned above, the purpose of rescaling is to change the initial baseline real and nominal yield curve so that the changed curve would be in line with a given target while maintaining the risk-neutral characteristics of the Base ESG file as demonstrated by the martingale property.

While this rescaling certainly is not new and has been widely used by a number of practitioners over the last 15 years, it has not been broadly publicised. Therefore, we are going to quote the main rescaling formulae here—let us begin by considering the formulae for bonds and the stochastic discounting factors:

$$\overline{ZCB}_T(t) = ZCB_T(t) \times \frac{ZCB_t(0)}{\overline{ZCB}_t(0)} \times \frac{\overline{ZCB}_{T+t}(0)}{ZCB_{T+t}(0)}$$

$$\overline{Disc}_t = Disc_t \times \frac{\overline{ZCB}_t(0)}{ZCB_t(0)}$$

where:

ZCBT (t) denotes the price of the zero coupon bond of term T at time t

Disct denotes the discount factor at time t and terms with a bar denote rescaled terms.

One can directly prove that the bond martingale property would hold for rescaled terms if it had been in place for the Base ESG file simply by making use of the formulae given above.

For equity indices, rescaling is carried out using the following formula:

$$\overline{Index}(t) = Index(t) \times \frac{ZCB_t(0)}{\overline{ZCB}_t(0)}$$

Similarly to the above, one can directly prove that the equity index martingale property would hold true for rescaled terms if it had been in place for the Base ESG file.

Furthermore, one can rescale such information as real yields or exchange rates, too. However, volatilities—notably, fixed interest (FI) volatilities—do not lend themselves to a rescaling approach because they are a function of yield curve dynamics, which have been already prescribed via the formulae given above. In fact, even a rescaling carried out in order to reflect a univariate nominal yield stress would lead to some (unintended) changes in FI volatilities.

Hence, we cannot hope to meet user-defined targets for FI volatilities by merely using the rebasing techniques quoted above. We need an additional step in our ESG Rebase process.

2.2 REWEIGHTING

Typically, all scenarios in a risk-neutral valuation ESG file used by an insurer are tacitly assumed to be of equal weight 1/N, where N is the number of simulations in the ESG file.

The gist of reweighting is to relax the above paradigm and 'just' assume that the weights are positive and their sum is 1. The newly won N-1 degrees of freedom can be then used in order to make the scenario file better comply with calibration targets such as FI volatilities, equity volatilities or martingale test results. Note that reweighting would not change anything at all in the rescaled scenarios produced in our previous step.

This approach was first introduced in finance almost 20 years ago by Avellaneda[1] while applications in a life insurance context have been discussed by Hoerig and Wechsung.[2]

Mathematically, the optimal weights are obtained as a solution to the following minimisation problem:

$$H = S(\{w_i\}) + \mu\left(\sum_{i=1}^{N} w_i - 1\right) + \sum_{m=1}^{M} \lambda_m \times \left(\sum_{i=1}^{N} \frac{w_i \times \overline{PV}_{m,i}}{\overline{P}_m} - 1\right)^2$$

Here, the following notation is used:

- Scenario index *i* runs from 1 to *N*
- Index *m* runs through calibration targets
- $w_i \geq 0$ denotes the weight of scenario *i*
- *S* is a measure of entropy (see below)
- μ is a Lagrangian multiplier
- λ_m is the importance weight for target *m*
- $\overline{P}_m$ is the rebased price* of target asset *m*
- $\overline{PV}_{m,i}$ is the rebased discounted present value of target asset *m* in scenario *i*

* 'Rebased price' means a price consistent with the rebased market conditions such as yield curve and volatilities. In the case of a martingale test target, the rebased price just means the perfect martingale test outcome of 1.

The first component of the above function measures entropy according to Kullback and Leibler,[3] or by how far the weights are away from the uniform weight set:

$$S(\{w_i\}) = \sum_{i=1}^{N} w_i \times \ln w_i$$

For the uniform weight distribution, the entropy functional attains its minimum:

$$S_{\min} = -\ln N$$

[1] Avellaneda, M. et. al. (2001). Weighted Monte Carlo: A new technique for calibrating asset-pricing models. Intl Journal of Theoretical and Applied Finance.
[2] Hoerig, M. & Wechsung, F. (2013). Scenario reweighting techniques for the quick recalibration of pricing scenarios, Der Aktuar.
[3] Kullback, S. & Leibler, R.A. (1951). On information sufficiency. Annals of Mathematical Statistics.

The second component of the function *H* ensures via the Lagrangian multiplier approach that the sum of weights is 1.

Finally, the third component of the function above is a squared error term which aggregates weighted squared deviations between rebasing targets (such as a FI volatility or a martingale test outcome) and their respective realisations. The weighting of the squared deviations applied via the importance weight parameters λ_m is useful to prioritise some targets above others and/or suppress some targets (for instance, targets for a second economy could be given importance weights of 0 if only one economy were relevant).

We conclude this section by providing some additional comments on the entropy term. The importance of this term stems from the need to avoid solutions where only a handful of simulations would feature significantly positive weights.

Yet the entropy score itself does not convey any intuitive message to the users of ESG Rebase. A much more intuitive measure for how far away a given weight set is from uniformity is provided by a related measure called the *effective number of scenarios*. This measure is derived from the entropy term as follows:

$$N_{eff} = e^{-S(\{w_i\})}$$

The effective number of scenarios displays, in an intuitive way, how difficult the rebasing targets have been to attain. A score close to the number of scenarios in the Base ESG file indicates little adjustment has been required to satisfactorily resolve the optimisation. A lower score indicates that a less uniform (more adjusted) weight scheme has been required.

3. ESG Rebase examples

In the previous section, we have introduced the two steps of the ESG Rebase algorithm from a theoretical viewpoint. In this section, we illustrate it by virtue of examples. For this purpose, we are going to discuss one example per step of the algorithm.

The risk-neutral Base ESG file used in the examples presented in this section and the following one contains 1,000 scenarios and has been generated using the cloud-based ESG, Milliman CHESS™.[4]

3.1 RESCALING EXAMPLE

As an example of classic rescaling, we consider an upward shift of 100 basis points (bp) to the nominal yield curve. Under rescaling the targeted shifted yield curve is exactly attained, so the purpose of this example is to verify that the rescaling indeed preserves the martingale property.

The table in Figure 2 shows how the martingale property has been satisfied by the Base ESG file for various bond terms and projection periods.

FIGURE 2: MARTINGALE PROPERTY SATISFIED

Results	Projection Period (t)												
Bond Term (T)	2.00	3.00	4.00	5.00	7.00	10.00	12.00	15.00	20.00	25.00	30.00	35.00	40.00
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	1.001	1.002	1.000	0.990
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.001	1.002	0.999	0.988
5.00	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	1.000	1.002	1.002	0.995	0.985
7.00	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.998	1.000	1.003	1.002	0.992	0.982
10.00	1.000	1.000	1.000	1.000	0.999	0.998	0.998	0.998	1.001	1.004	1.002	0.988	0.978
20.00	1.001	1.001	1.001	0.998	0.998	0.996	0.996	0.995	1.005	1.008	1.001	0.971	0.969
30.00	1.002	1.002	1.001	0.997	0.996	0.994	0.993	0.994	1.009	1.010	0.999	0.962	0.965

Following the rescaling, we compute the same martingale statistics. The differences between the original results and those for the rescaled file are shown in the table in Figure 3.

FIGURE 3: ORIGINAL RESULTS VS. RESCALING RESULTS

Results	Projection Period (t)												
Bond Term (T)	2.00	3.00	4.00	5.00	7.00	10.00	12.00	15.00	20.00	25.00	30.00	35.00	40.00
2.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

CONCLUSION

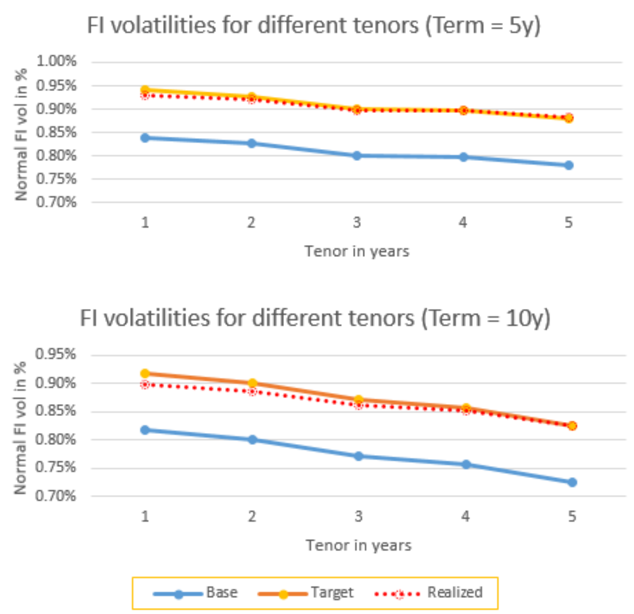
We observe that the original and rescaled results are the same as expected.

3.2 REWEIGHTING EXAMPLE

As discussed, we apply reweighting in ESG Rebase because we have to be able to accurately attain user-defined volatility targets. Hence, a natural reweighting example is a FI volatility shock.

Given the current low-yield environment, we measure FI volatilities using the normal volatility convention rather than the Black convention. As our sample FI shock, we consider a flat upward 0.1% normal volatility shock. The diagrams in Figure 4 (one for the five-year term, the other for the 10-year term) illustrate how well this target has been attained

FIGURE 4: FI VOLATILITIES FOR DIFFERENT TENORS, 5-YEAR TERM AND 10-YEAR TERM



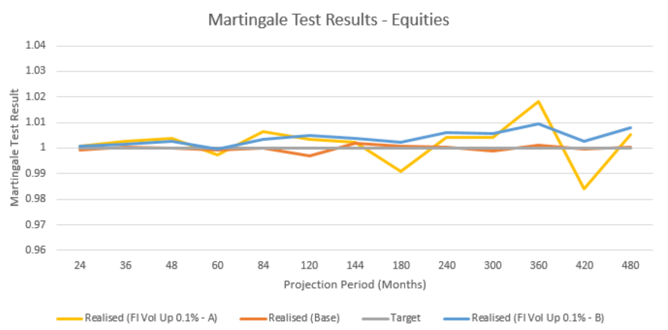
[4] For more information on Milliman CHESS, please see http://www.milliman.com/Solutions/Products/Milliman-CHESS-Cloud-Hosted-Economic-Scenario-Simulator/

We can see that reweighting delivers a scenario set which exhibits FI volatilities very close to those required.

We have to also keep an eye on the martingale tests. The diagram in Figure 5 illustrates how the martingale test results for equities under reweighting compare to the corresponding results for the Base ESG file. In Figure 5, we display a four-way comparison as follows:

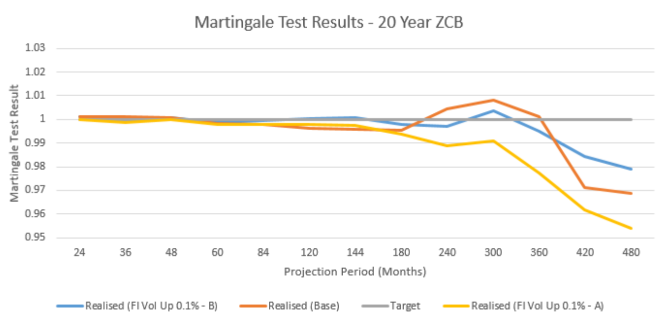
- The grey line represents the theoretical target of 1.
- The orange line shows the martingale results for the Base ESG file.
- The yellow line shows the martingale results for our FI volatility shock produced by ESG Rebase when we allocate moderate importance weights to the martingale targets (Reweighting Exercise A).
- The blue line illustrates the martingale results for the same FI volatility shock produced by ESG Rebase when we assign higher importance weights to the martingale targets (Reweighting exercise B).

FIGURE 5: MARTINGALE TEST RESULTS, EQUITIES



Now we have to examine the martingale tests for bonds of different terms to maturity. As an illustration, Figure 6 shows how the martingale test results for the 20-year zero coupon bond (ZCB) under reweighting compare to the corresponding results for the Base ESG file.

FIGURE 6: MARTINGALE TEST RESULTS, 20-YEAR BONDS



We observe that the target-specific importance weight parameters in ESG Rebase give us the flexibility to control the martingale results. It is important to note that the weight scheme refinement still delivers nearly identical FI volatilities in both ESG Rebase exercises and only incurs a small reduction in the effective number from 877 in Exercise A to 854 in Exercise B. Recall that the Base ESG file has 1,000 simulations, so both these scores represent very good outcomes given the significance of the 0.1% normal volatility up shock. For a comparison, if we apply the complete ESG Rebase approach (including reweighting) to the 100bp nominal yield up shock from the previous example, the effective number of scenarios is 985—note that it is much easier to keep FI volatilities constant than to significantly modify them.

CONCLUSION

By using reweighting, we can attain ambitious volatility shock targets while maintaining very good martingale test outcomes and without departing too far from weight uniformity.

4. Proxy modelling

In this section, we illustrate how ESG Rebase performs in the context of an end-to-end proxy modelling process featuring the following components:

- Produce rebased scenarios for Least Squares Monte Carlo (LSMC) proxy modelling*
- Run cash flow model on rebased scenarios thousands of times to produce LSMC regression input data
- Calibrate LSMC curves
- Produce rebased risk-neutral scenario files for a range of out-of-sample validation points
- Run cash flow model on these validation points
- Produce LSMC goodness-of-fit reports

* The generation of rebased scenarios for LSMC proxy modelling purposes can be thought of as a two-step approach. Firstly, apply the usual ESG Rebase algorithm to produce *N* risk-neutral scenarios starting at a given outer point from the *N* risk-neutral scenarios in the Base ESG file. Secondly, sample the necessary number of inner simulations (e.g., 2 or 10 per outer point) from the *N* rebased scenarios above, without introducing any bias.

4.1 EXAMPLE #1: SIMPLE ASSETS

Our first end-to-end proxy modelling example is a portfolio of 30 equity put options featuring different characteristics (in-, at- and out-of-the-money, different terms).

The valuation of these options in the reference model is carried out using a closed-form (Black-Scholes) approach. Our aim is to calibrate an LSMC polynomial representing the market value of the put option portfolio as a function of the following five risk drivers:

- Nominal yield principal components (PC) 1-3
- Equity index (underlying)
- Equity volatility

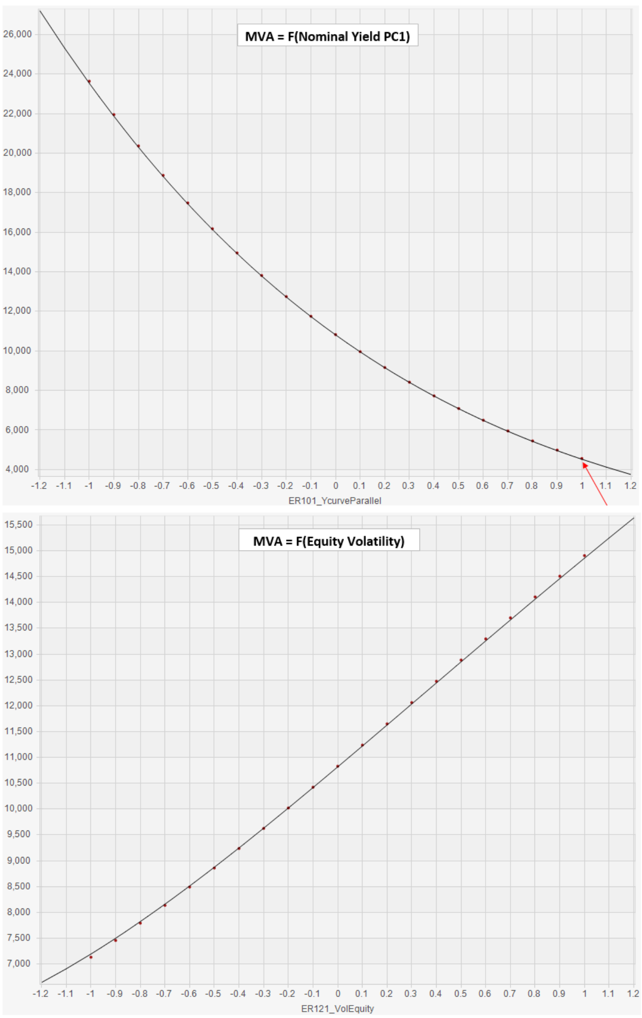
We use ESG Rebase in order to generate scenarios for the calibration of our LSMC polynomial—a total of 25,000 outer points—different realisations of the above risk drivers.

Because a closed-form valuation of an option would only depend on the valuation point (that is, the values of the above risk drivers) and not on any scenario dynamics after the valuation point, we can restrict ourselves to a deterministic LSMC calibration in this example—that is, we only need the certainty-equivalent scenario for each outer point. In other words, our LSMC calibration budget in this example amounts to 25,000 x 1 = 25,000 simulations.

Given that all training points and validation points are produced using the same Black-Scholes closed-form valuation approach, we have to expect an excellent goodness-of-fit from our LSMC calibration in this example.

Indeed, we do obtain a very good fit for the market value of assets (MVA) as illustrated by the two plots in Figure 7. Please note that the risk driver realisations have been normalised to lie in the range [-1.2, +1.2]. Because results for 1-in-200-year shocks have to be reliable given their importance for Solvency II purposes, the 1-in-200-year shocks must be well inside the calibration data cube rather than at its boundary. In our examples, we calibrate LSMC curves by going 20% beyond the 1-in-200 shocks in each dimension.

FIGURE 7: LSMC FIT FOR SIMPLE ASSETS



Note that the red arrow in the first plot in Figure 7 highlights the position of the upward 1-in-200-year shock to the first principal component of nominal yields, while the red points represent validation results produced by the reference model (a Black-Scholes Excel workbook).

4.2 EXAMPLE #2: SIMPLE LIABILITIES

Our second end-to-end proxy modelling example is a portfolio of 30 unit-linked contracts with embedded maturity guarantees featuring different characteristics (in-, at- and out-of-the-money, different terms) similar to the equity put options considered in the example above. For the purposes of this example, decrements are ignored.

The valuation of these contracts in the reference model is carried out using a stochastic valuation approach. Our aim is to calibrate an LSMC polynomial representing the market value of the put option portfolio as a function of the same five risk drivers as in the example above.

We use ESG Rebase in order to generate scenarios for the calibration of our LSMC polynomial—a total of 25,000 outer points—different realisations of the above risk drivers. However, this time we allow for 10 inner simulations for each outer point given the stochastic nature of the LSMC calibration needed in this example. In other words, our LSMC calibration budget in the current example amounts to 25,000 x 10 = 250,000 simulations.

Our LSMC calibration fit for this example is illustrated by the two plots in Figure 8. As above, the red points represent validation results produced by the reference model (a traditional actuarial stochastic cash flow model in this example). Note that each of these validation points has been generated using 1,000 risk-neutral scenarios produced by appropriately rebasing the Base ESG file to the respective validation point.

FIGURE 8: LSMC FIT FOR SIMPLE LIABILITIES

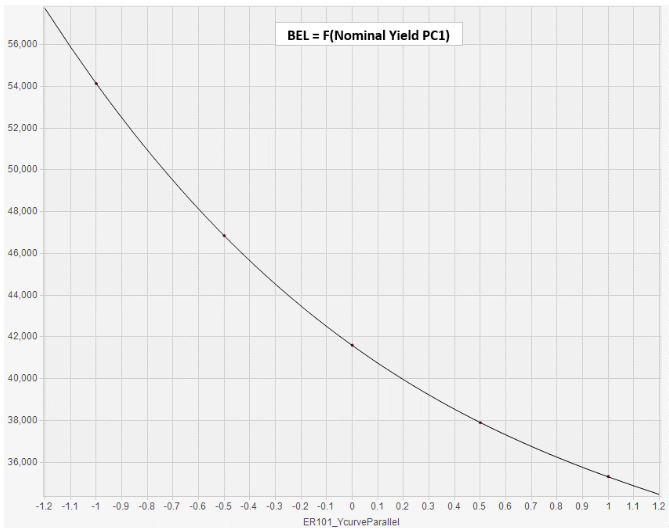
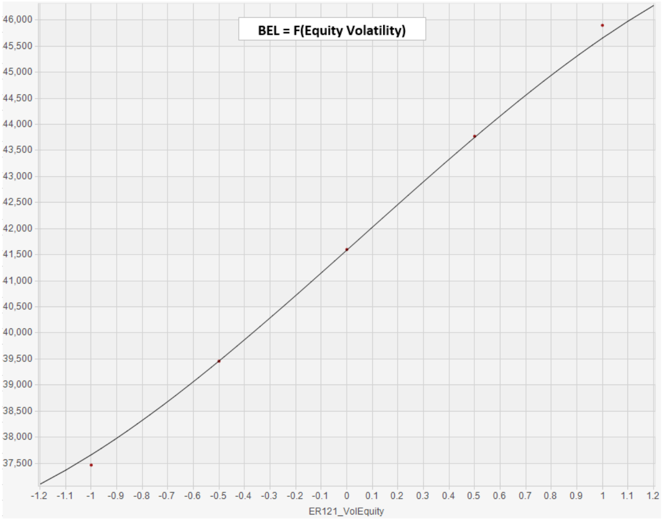


FIGURE 8: LSMC FIT FOR SIMPLE LIABILITIES (CONTINUED)



CONCLUSION

We obtained an excellent fit again with average deviations amounting to 0.0%. Yet a critical reader could point out that the optionality contained in a typical insurance liability portfolio is often path-dependent while the optionality considered in our tests so far is not. Hence, in our final example, we will consider path-dependent options.

Note that the Best Estimate Liability (BEL) above not only includes the embedded guarantees but also the underlying fund value of 1,000 for each of the 30 contracts.

4.3 EXAMPLE #3: COMPLEX LIABILITIES

Our final end-to-end proxy modelling example is a portfolio of 30 unit-linked contracts with embedded guarantees, which are now annually reset: to be more precise. In each projection year, the strike is set at-the-money to the fund value prevailing at the start of the year. As in the example above, decrements are again ignored.

From the viewpoint of financial mathematics, such embedded guarantees are nothing else than Cliquet put options. We demonstrate how they work by virtue of two illustrative examples in Figure 9.

FIGURE 9: EMBEDDED GUARANTEES EXAMPLES

Example A:

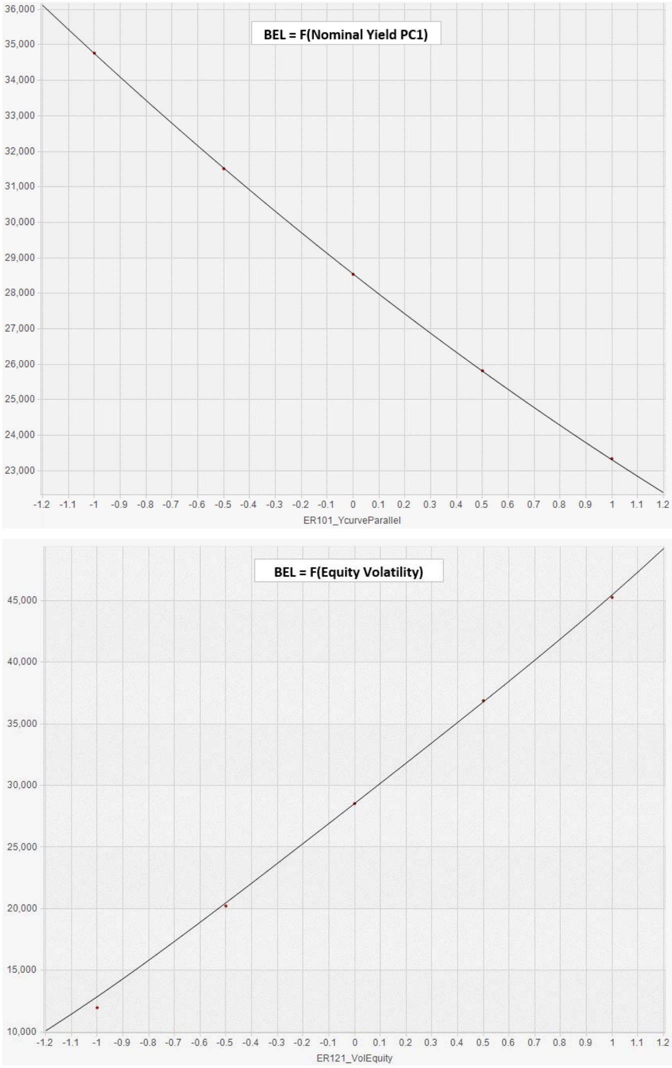
	0	1	2	3	4
Fund	1000.00	405.47	367.50	371.94	376.00
Strike		1000.00	405.47	367.50	371.94
Cashflow		594.53	37.97	0.00	0.00

Example B:

	0	1	2	3	4
Fund	1000.00	705.95	687.30	479.76	361.16
Strike		1000.00	705.95	687.30	479.76
Cashflow		294.05	18.65	207.54	118.60

The LSMC calibration and LSMC validation in this example are carried out similarly to the example above. In analogy to the example above, let's consider our usual validation plots, in Figure 10.

FIGURE 10: LSMC FIT FOR COMPLEX LIABILITIES

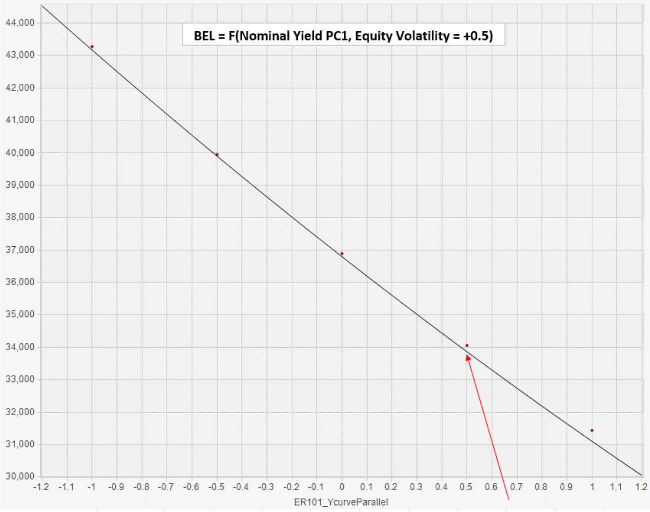


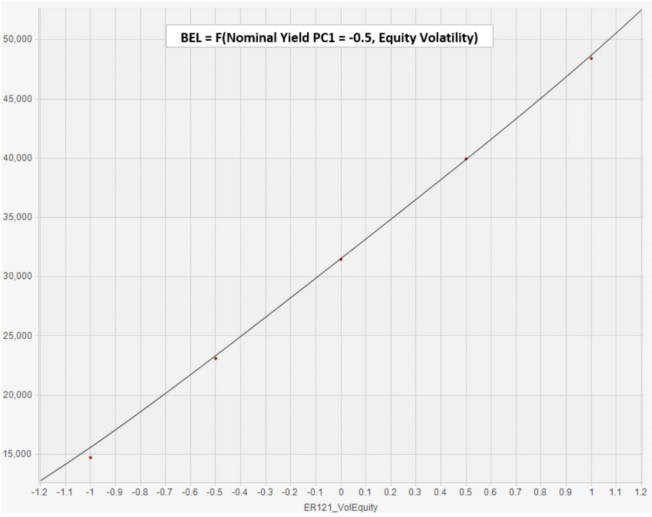
While these plots do illustrate a good fit, it is worth pointing out that—so far—all our plots have illustrated the fit at points of the risk driver space featuring a shock to just one risk driver with all the other risk drivers kept at their base values. This raises the question whether or not the goodness-of-fit would be similar or maybe worse if we considered combined shocks—relevant, e.g., for Solvency II work—as our validation points. In order to look into this question, we are now going to consider validation points relative to the following combined shock position rather than relative to base: Nominal Yield PC1 = -0.5 and Equity Volatility = +0.5.

In other words, the above position represents a combination of 50% of the downward 1-in-200-year shock to nominal yield PC1 and 50% of the upward 1-in-200-year shock to equity volatility. Let us examine the goodness-of-fit of our LSMC curve for combined shocks in these two dimensions, illustrated in

Figure 11. Note that again the red arrow in the first plot in Figure 11 points to the combined shock, consisting of 50% of the upward 1-in-200-year shock to the first principal component of nominal yields and 50% of the upward 1-in-200-year shock to the equity volatility.

FIGURE 11: LSMC FIT FOR COMPLEX LIABILITIES, CONTINUED





CONCLUSION

For our portfolio of path-dependent options, we obtained a very good fit. The average deviation across our univariate and combined validation points amounted to 0.9%. This average deviation is larger than those observed in the previous sections, which is logical given the much more complex nature of path-dependent options. That said, we conclude that an end-to-end process relying on ESG Rebase and Least Squares Monte Carlo can generate a very good fit on a portfolio of path-dependent options which are typical for a life insurance book.

5. Outlook

We have seen how ESG Rebase can be successfully applied to a range of actuarial modelling use cases ranging from producing risk-neutral ESG files for various stresses to generating many thousands of scenarios needed to calibrate a Least Squares Monte Carlo proxy model, e.g., for Solvency II internal model purposes.

Needless to say, the use cases for ESG Rebase go well beyond work related to Solvency II. For example, IFRS 17 will require production of risk-neutral valuations of a large number of insurance cohorts (e.g., annual) using the nominal yield curve (and FI volatility surface) as per cohort inception. ESG Rebase can consistently produce such a large number of required ESG files from the usual Base ESG file as a common data source and, of course, it can do so automatically.

Last, but not least: It is also worth noting what ESG Rebase cannot produce. Because the rebased scenarios are always generated by using the same rescaling formulae given in Section 2 above, ESG Rebase cannot produce scenarios in line with any particular probability distribution. Therefore, ESG Rebase cannot be used to rebase real-world scenario sets.

6. Literature

[1] Avellaneda, M. et. al. (2001). Weighted Monte Carlo: A new technique for calibrating asset-pricing models. Intl Journal of Theoretical and Applied Finance.

[2] Hoerig, M. & Wechsung, F. (2013). Scenario reweighting techniques for the quick recalibration of pricing scenarios. Der Aktuar.

[3] Kullback, S. & Leibler, R.A. (1951). On information sufficiency. Annals of Mathematical Statistics

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