Milliman Research Report

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SEPTEMBER 2014

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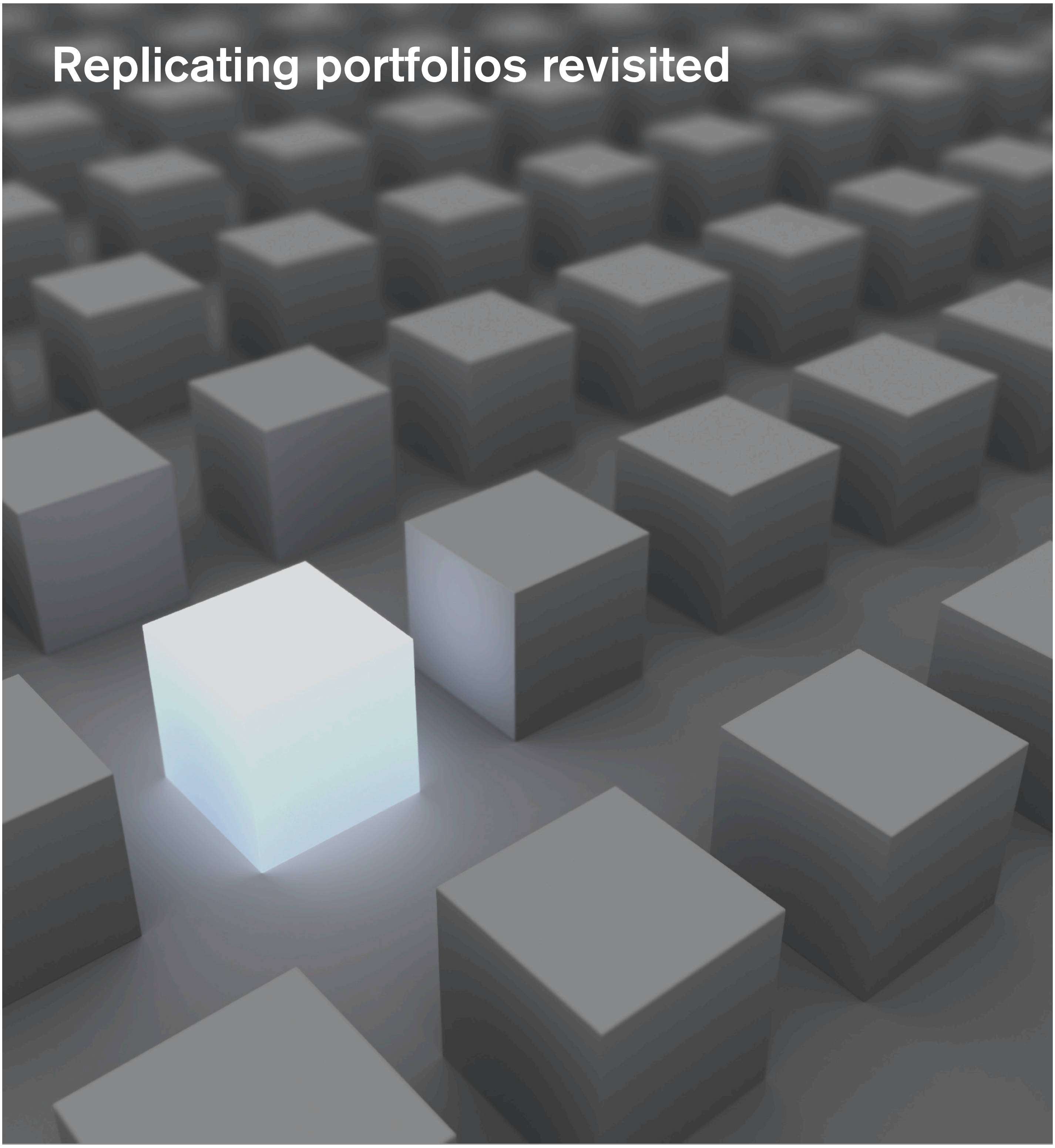
Replicating portfolios revisited



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1 STOCHASTIC MODELING AND LIABILITIES AT T=1 PROBLEM

Recent years have seen an increasing sophistication in the risk management regulatory frameworks for insurance companies around the world, like Solvency II in Europe or the National Association of Insurance Commissioners' Own Risk and Solvency Assessment (NAIC ORSA) in the United States.

Many market leaders in the insurance industry use stochastic projections for their risk-based capital (RBC) calculations instead of using a standard formulaic way to calculate the RBC.

Deriving the market value of liabilities in a case where the liabilities have options and guarantees embedded, one has to use a market-consistent, risk-neutral valuation. However, this calculation is rather complex and, when performed in a straightforward manner, involves running a cash-flow projection model under a large number of risk-neutral scenarios. When performing stochastic projections of the company value for RBC purposes, one has to generate tens of thousands of real-world scenarios for a given time horizon, and at the end of this time horizon one needs to calculate the market values of liabilities. Even if the computation of the liabilities' market value for a single scenario is relatively fast, one can quickly incur very large processing and calculation costs, as this single computation needs to be repeated tens of thousands times.

This situation calls for a time- and money-saving solution, and several proxy techniques have been proposed in recent years, including replicating portfolios, curve fitting and least squares Monte Carlo techniques. Here, the RBC calculation relies on approximation functions, where results for market values of liabilities are obtained easily. However, coming up with the approximation function is a tedious task in itself and requires some effort. In this paper we will take a deeper look at the replicating portfolios method and how it can be enhanced to achieve better solutions to the approximation problem.

2 CHALLENGES AND KNOWN ISSUES FOR REPLICATING PORTFOLIOS

For the fundamentals of replicating portfolios (RP) we refer to [1] and [2]. The overall idea of the RP approach is to find a suitable set of candidate assets that explain the behaviour of the liabilities—i.e., replicate the liability values—for a variety of market conditions. To be more precise and formal, this task is twofold:

(i) Determination of the candidate assets explaining the behaviour of the liabilities

(ii) Calibration of the portfolio, i.e., assessing appropriate volumes to the chosen candidate assets

We will now address the major issues and challenges in the context of the calibration of RP, using the setup introduced above (see also [2]).

(1) Robustness and stability: If the distance between the liability value and the portfolio value is measured by the quadratic norm, the calibration itself can be reduced to an ordinary least squares problem. However, any such optimization problem is known to be not robust in the sense that small changes of the input for the optimization might have a highly significant influence on the resulting RP.

(2) Correlation vs. causality: For determining whether a particular candidate asset is relevant for replicating the liabilities insurers typically assess the residual sum of squared errors R2 of a particular portfolio. This is equivalent to an assessment of the correlation between the liability values and values of the portfolio on the calibration scenarios. Therefore, they perform a calibration of the RP with and without the asset under consideration and judge whether the gain in R2 or correlation is significant. Therefore insurers face the classical 'correlation vs. causality' pitfall. In the case of RPs this means that assets which display a high level of correlation on the calibration scenarios cannot be distinguished and are hence substitutes for each other, even though their actual nature might be quite different.

(3) Coverage: A robust and reliable RP is supposed to replicate the behaviour of the liabilities for any kind of market movement. Therefore, for a successful calibration of the RP it is crucial to use a variety of different and joint market conditions to teach the RP how the liabilities will react on such a market movement.

Issues (1) and (2) are caused by the fact that the candidate assets display a high level of correlation on the calibration scenarios, while issue (3) is caused by a lack of variety in the calibration scenarios or candidate assets not picking up market movements. The first effect—highly correlated candidate assets on the calibration scenarios—can be split up in two separate causes:

I. Dependencies of risk factors: The risk factors are part of the economical scenario generator (ESG) and are modelled via appropriate capital market models. This includes yield curves, spread curves and indices, among others. A typical set of valuation scenarios displays a high degree of dependency between these risk factors since these risk factors, once calibrated to market conditions, possess a high dependency by their very nature:

   a. Zero-coupon bonds with different maturities tend to move in the same direction.

   b. Credit-risky assets share a major part of their value movement with their risk-free underlying portion.

   c. Means of the index movements are determined by the initial yield curve.

II. Dependencies of underlying assets: While it is clear that dependent risk factors inevitably result in a dependency of the assets related to these risk factors, we can, in addition to this, have dependencies between candidate assets even if the risk factors are perfectly independent:

   a. Call and put options with similar strikes or exercise dates are naturally dependent due to put call parity and can replicate each other near perfectly.

   b. Swaps can be considered as combinations of zero-coupon bonds.

   c. Any kind of option with only a few out-of-the-money paths is highly correlated to its underlying.

Solving these issues inevitably requires addressing the choice of calibration scenarios and the selection of appropriate assets. Both are covered in the following section.

3 RP IMPROVEMENTS

3.1 Calibration space definition

This subsection addresses specifically the potential dependency between the risk factors. The first and perhaps one of the most challenging tasks when calibrating an RP is the choice of the calibration scenarios. At present, insurers typically use few sets of sensitivities with at least 1,000 valuation scenarios each. The approach we are suggesting in this subsection can be regarded as a consequent further development of the basic approach of using few sensitivities:

We suggest to consider a large number (e.g., 1,000) of sensitivities (we will call the initial conditions of the sensitivities outer scenarios) with orthogonal risk factor movements and only a few (e.g., five) valuation scenarios as continuation (we will call these risk-neutral scenarios based on one outer scenario inner scenarios) per outer scenario. With 'orthogonality' we refer to two distinguished properties:

- All risk factors are stressed simultaneously but independently from each other.

- All risk factors are stressed in a uniform way.

In practice this means that for each risk factor we identify its minimum and maximum value under consideration and generate the outer scenarios by sampling within these bounds simultaneously
for each of the factors.

The strengths of this approach are as follows:

- Independency of risk factors: Intrinsic to this technique the single risk factors are independent, being independently and uniformly simulated within their bounds.

- Coverage of risk factor space and bias: Following this approach yields an even level of 'illumination' of the risk factor space. Therefore, the resulting RP has no bias in contrast to the simple approach described above, which can replicate the liabilities for some market conditions particularly well for the price of lacking replicating quality for other conditions.

- Scalability: Since there is no bias towards a particular area of market conditions, the overall number of calibration scenarios used for the replication is perfectly scalable.

3.2 Factor space selection

Even though the risk factors are uncorrelated as introduced in the previous subsection, this does not necessarily imply that candidate assets such as bonds with different maturities are also uncorrelated.

Hence, we are now in the situation as described in Section 1, where there is a large set of potentially correlated candidate assets. To overcome this, we have two options:

- Use a subset of candidate assets for the calibration, where the subset is chosen so that it does not contain multicollinearities
- Perform a so-called principal component (PC) regression

As the first approach does not require further explanation, we will turn our attention to the second one for the remaining part of this subsection.

The idea of the PC regression is to use synthetic assets as explanatory variables instead of the explanatory variables themselves. These synthetic assets are the principal components of the set of candidate assets on the calibration scenarios. The main advantage of using synthetic PC assets is that these are uncorrelated by construction.

Technically speaking, we transform the set of candidate assets into a set of synthetic assets which are uncorrelated on the calibration scenarios, reformulate the optimization problem in terms of these assets, solve it and retransform the solution into candidate assets. Therefore, the choice of the PCs used for the regression analysis is a task of major importance. On the one hand, one wants to eliminate large variances due to multi- collinearities (i.e., deleting the PCs with low variances), but, on the other hand, it is not desirable to delete the PCs that have a large correlation to the liabilities.

We suggest using likelihood-based criteria like Akaike's information criterion (AIC) or Bayesian information criterion (BIC) in combination with stepwise model selection procedures to determine the subset of PCs that should be included in the regression. Therefore, we start with choosing the PC with highest correlation to the liabilities. In the next steps we gradually add the principal component that leads to highest correlation between portfolio value and liability value. In order to avoid overfitting, we finish this procedure according to the above information criteria.

The benefits of this approach can be summarized as follows:

- Robustness: By calibrating (uncorrelated) principal components of the candidate assets, we transform the optimization problem into a robust framework.
- Objectivity: Using stepwise model selection procedures and assessing information-based criteria removes any kind of subjectivity in the model selection process.

4 CASE STUDY

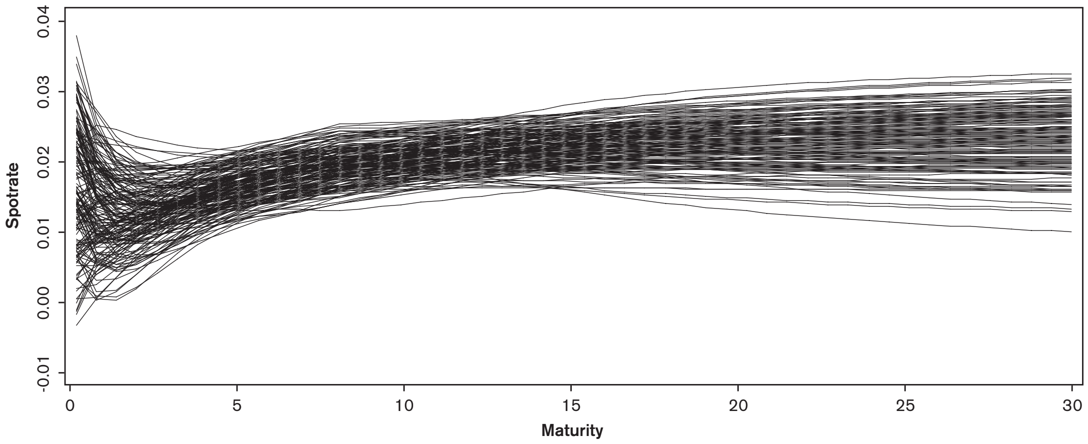
This case study demonstrates the power of the techniques described in the previous sections.
The overall set up is as follows:

- The set of candidate assets consists of risk-free zero-coupon bonds with different maturities as well as swaps, swaptions, index call options with different strikes, maturities and tenors.
- The liabilities consist of a subset of these candidate assets scaled with proper volumes to make the overall liability value and its distribution realistic. We also add some Gaussian noise to the liabilities, so that we cannot perfectly replicate them.

Under our real-world view the true mean value of the liabilities at $t = 1$ is -439.23 millions while the respective 99.5% quantile is -454.57 millions, yielding an RBC of 15.34 millions. In the following we calibrate an RP for these liabilities in different ways (calibration has been performed by Milliman STAR solutions NAVI®) and use the resulting quality measures to benchmark the quality of the replication. Our overall scenario budget for the calibration of the RP is 3000. We distinguish two sets of calibration scenarios:

- Non-orthogonal calibration scenarios (produced by GEMS® Economic Scenario Generator (ESG)): Three initial yield curves with 1,000 market-consistent valuation scenarios
- Orthogonal calibration scenarios: 1,000 initial yield curves derived by joint stresses of the first four principal components of historical yield curve movements (cf. Figure 1) with three market-consistent valuation scenarios (produced by GEMS® ESG)

FIGURE 1: ORTHOGONAL CALIBRATION SCENARIOS: INITIAL YIELD CURVES



Next, we perform three different calibrations of the RP, gradually including the techniques introduced previously to demonstrate their impact on the overall calibration:

Case 1: Calibration of an RP using 3,000 non-orthogonal calibration scenarios and choosing the most significant set of candidate assets for the replication via a model selection procedure

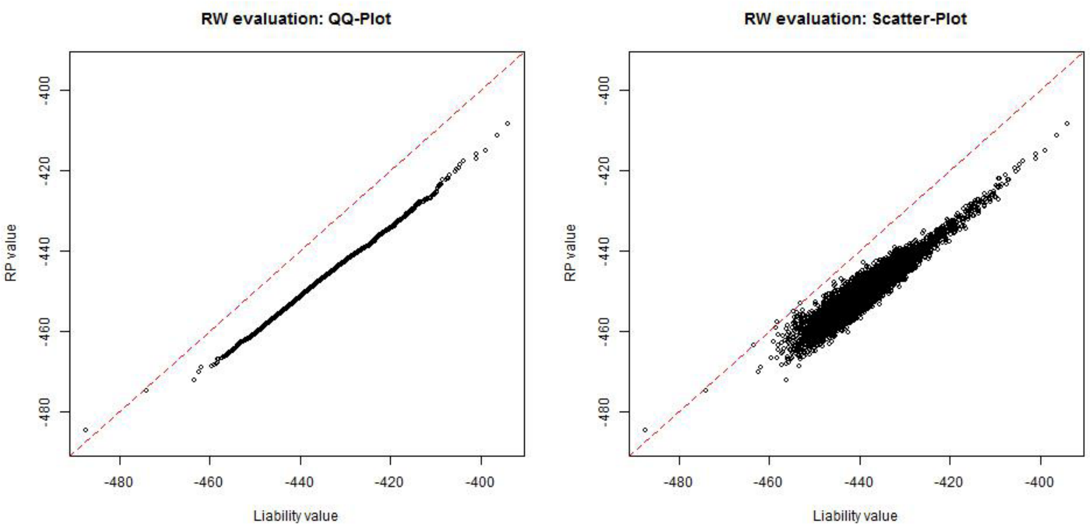
Case 2: Calibration of an RP using the 3,000 orthogonal calibration scenarios and applying a principal component analysis on the set of candidate assets together with a model selection procedure

4.1 Case 1

In this subsection, we fit an RP which consists of a (not pre-specified) number of candidate assets for the non-orthogonal calibration scenarios. We start with one asset and gradually add the candidate assets that lead to the highest increase of the R2 between liability value and the corresponding value of the RP. The process is stopped when the AIC does not increase anymore. According to AIC, we choose for the non-orthogonal calibration scenarios 19 assets.

While the fit on the calibration scenarios is very good (R2 = 99.88%), there is a rather high deviation between the value of the RP and the liability value on the out-of-sample real-world scenarios (cf. Figure 2—here, the RP value is always much lower than the true liability value). The resulting mean of the RP at t = 1 is −450.88 millions and the 99.5% quantile −464.56 millions yield an RBC of 13.68 millions–a value which is 10.8% lower than the true RBC, and thus the true RBC is substantially underestimated.

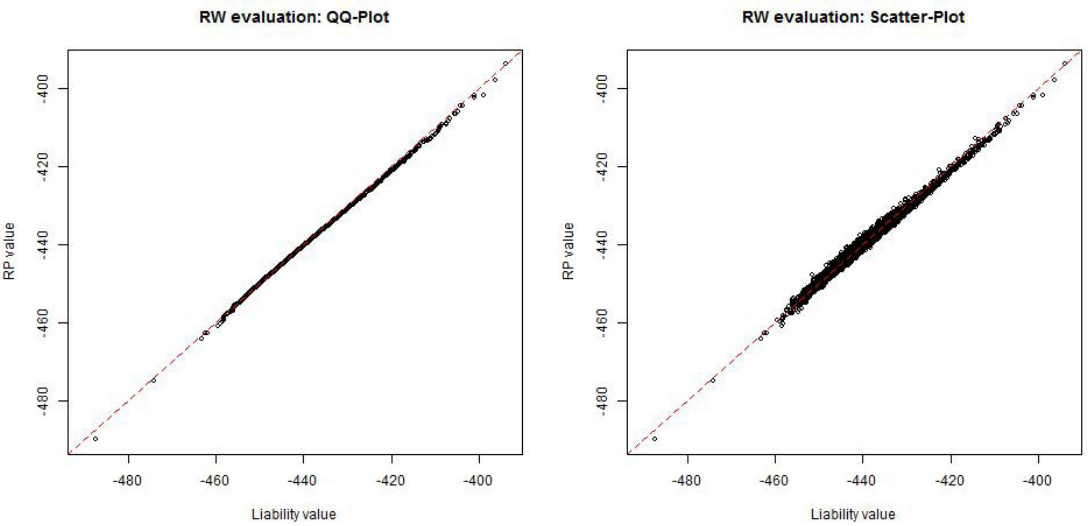
FIGURE 2: SCATTER- AND QQ-PLOT OF THE RP WITH 19 ASSETS FOR NON-ORTHOGONAL CALIBRATION SCENARIOS – REAL WORLD



4.2 Case 2

Here, we fit an RP which consists of a number of principal components of the candidate assets. For the calibration of the RP we use the orthogonal calibration scenarios. We use the stepwise model selection procedure as described in the previous section. According to AIC, we choose 47 PCs. The fits on the calibration scenarios (R2 = 99.86%) and on the real-world scenarios (cf. Figure 3) are almost perfect. The resulting mean of the liabilities at t = 1 is –439.76 millions and 99.5% quantile –454.92 millions yield RBC 15.16 millions–a value which is only 1.2% lower than the true RBC.

FIGURE 3: SCATTER- AND QQ-PLOT OF THE RP WITH 47 PCS OF THE CANDIDATE ASSETS FOR ORTHOGONAL CALIBRATION SCENARIOS - REAL WORLD



When comparing the results for these two cases, we can clearly see that the resulting quality is increased by using non-orthogonal calibration scenarios and principal component regression: It is remarkable that the in-sample validations, i.e., the scatter and QQ-plots as well as the R2 figure on the calibration scenarios are nearly identical for both cases. However, the fact that the real-world–i.e., out-of-sample–scatter- and QQ-plots and the resulting quantiles differ significantly indicates that the calibration on the non-orthogonal scenarios was neither robust nor reliable and displayed a smaller degree of replicating quality.

5 SUMMARY

We introduced two methods allowing for a robust and reliable calibration of replicating portfolios:

- Use of orthogonal calibration scenarios to get a broad and even coverage of the risk factor space with independent risk factors and a scalable number of calibration scenarios

- Application of principal component analysis of the candidate assets in combination with a stepwise, criteria-based model selection procedure to sub-divide the portfolio into uncorrelated building blocks

We showed the merits of these methods and recommended the use of orthogonal calibration scenarios together with principal component analysis of the candidate assets when calibrating an RP. In our case study, we showed the dangers of non-orthogonal calibration scenarios and the merits of using orthogonal calibration scenarios and PCs (linear combinations of the candidate assets) when applied to RBC values.

6 REFERENCES

[1] J. Oechslin et. al., 2007, Replicating Embedded Options, Life & Pension Risk, pp. 47-52.

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