Milliman Research Report

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The new normal

Using the right volatility quote in times of low interest rates for Solvency II risk factor modelling

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TABLE OF CONTENTS

INTRODUCTION

A common way of quoting a swaption price is referring to its 'implied volatility' rather than the price itself. The implied volatility for the swaption is the volatility parameter required in a benchmark pricing model, which allows for closed-form prices, for the modelled price to replicate the market price of the swaption. The benefit of this method of quotation is that it removes the effect of the parameters not related to volatility that are contributing to the swaption price, such as the underlying yield curve, its strike, maturity, and tenor. Implied-volatility quotation therefore allows for a comparison of swaption prices among all different swaptions and over time.

For insurance companies writing interest-rate-related guarantees and options, the swaption implied volatility is a particularly relevant risk factor. Swaption implied volatility is a measure of the market volatility of the interest rate yield curve, which directly affects the costs of these interest rate guarantees and options. Therefore, for both valuation and risk models used to assess the costs of these guarantees and options, insurance companies rely on these quotes.

Typically many insurance companies have been using the Black model as a benchmark pricing model to derive the implied volatility quote, which is thus often referred to as Black volatility. The interest rate movements for the euro in the past six to nine months, however, have unveiled a major drawback of the Black volatility quote, which can affect current best practice approaches of insurance companies' risk and valuation models in a significant way. This white paper analyses and explains the challenges facing the Black model in the current interest rate environment and introduces an alternative model to address them. The paper concludes by demonstrating which parts of insurance companies' risk and valuation models are affected.

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Implied volatility quotes remove the effect of the parameters not related to volatility and hence allow for a comparison of swaption prices among different swaptions and over time.

INTRODUCING POTENTIAL QUOTATION MODELS

In this section we introduce the classic Black model as well as the so-called Normal model, which is our proposal for an alternative.[1] Both of them model the evolution of the forward swap rate $S_{[T_0,T_n]}(t)$ over time t for a payer interest rate swap[2] with maturity in T_0 and payments in $T_1<\dots<T_n$ $(T_0<T_1)$. Formally, we have

$$S_{[T_0,T_n]}(t) = \frac{P(t,T_0) - P(t,T_n)}{\sum_{i=1}^{n} P(t,T_i)}$$

where $P(t,T)$ is the value of a zero-coupon-bond with nominal 1, maturity T at time t.

The evolution of this forward swap rate is the key driver for the value of a swaption, i.e., an option to enter into this swap agreement at a predefined price.

The Black model

Here we briefly describe and discuss the Black model. [2] In this model, the evolution of the forward swap rate $S_{[T_0,T_n]}(t)$ for a swap with maturity in T_0 and tenor $T_n - T_0$ is modelled through the stochastic differential equation

$$\frac{dS_{[T_0,T_n]}(t)}{S_{[T_0,T_n]}(t)} = \sigma_{LN} dW(t)$$

with a Wiener process $W(t)$ and lognormal volatility σ_{LN}, i.e., the forward swap rate is assumed to be lognormally distributed as this equation describes the relative changes of the forward swap rate. The standard deviation of the relative changes in the forward swap rate is the constant Black volatility σ_{LN}. In other words, the forward swap rate is assumed to evolve according to geometric Brownian motion.

In the Black model the price of a payer swaption with strike K at time 0 can be determined analytically via

$$P_{payer,Black} = \left(S_{[T_0,T_n]}(0)\Phi(d_1) - K\Phi(d_2)\right)\sum_{i=1}^{n} P(0,T_i)\,,$$

where Φ is the cumulative distribution function of a standard normal variate, and

$$d_{1,2} = \frac{\log(S_{[T_0,T_n]}(0)/K) \pm \frac{1}{2}\sigma_{LN}^2 T_0}{\sigma_{LN}\sqrt{T_0}}\,.$$

The Normal model

The Normal model is an alternative benchmark model for swaption pricing and the derivation of implied volatility quotes. Here, the evolution of the forward swap rate $S_{[T_0,T_n]}(t)$ is given through the stochastic differential equation (SDE)

$$dS_{[T_0,T_n]}(t) = \sigma_N dW(t)$$

with a Wiener process $W(t)$ and Normal volatility σ_N, i.e., the forward swap rate is assumed to be normally distributed or evolves according to a standard Brownian motion. The standard deviation of absolute basis point (bps) changes of the forward swap rate is the constant normalised volatility σ_N. Under the Normal model, there is a nonzero probability for the forward swap rate being negative.

1 Note that we restrict ourselves in this paper to these two models; other models often quoted in this context are, e.g., the shifted version of the Black model or the 'constant elasticity of variance' model.
2 We confine ourselves to payer swaptions and refer to the well-known put-call-parity to obtain the values of the corresponding receiver swaptions.

The price of a payer swaption at time *0* can be determined analytically via

$$P_{payer,N} = \sigma_N \sqrt{T_0} \left(\widehat{d_1} \Phi(\widehat{d_1}) + \varphi(\widehat{d_1}) \right) \sum_{i=1}^{n} P(0, T_i)$$

where φ is the probability distribution function of a standard normal variate, and

$$\widehat{d_1} = \frac{S_{[T_0,T_n]}(0) - K}{\sigma_N \sqrt{T_0}}$$

[4]

Relationship between Black and Normal model

The simplest way to convert Black volatilities into Normal volatilities (and vice versa) is via the corresponding swaption prices, by determining the swaption price resulting from the Black/Normal volatility quote under consideration and determining the implied Black/Normal volatility via solving the corresponding pricing formula.

A useful approximation linking implied Black and Normal volatility quotes for at-the-money swaptions is

$$\sigma_{LN} \approx \frac{\sigma_N}{S_{[T_0,T_n]}}.$$

[3]

Note that this approximation is less accurate in the case of very low interest rates, i.e., once $S_{[T_0,T_n]}$ is close to zero.

WHICH MODEL TO FAVOR?

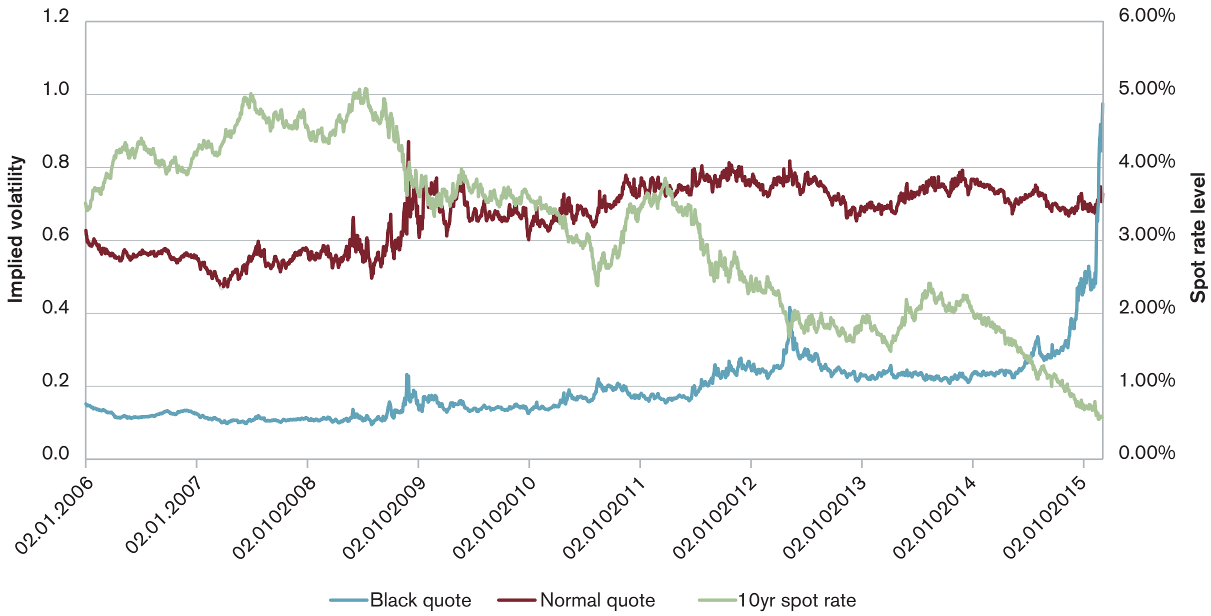
A look at the past

As we have seen, both the Black and the Normal models simulate the key driver for the swaption prices, the forward swap rate, in a very elementary way. While the Black model assumes a lognormal distribution, the Normal model gets its name from the normality assumption for the distribution of the forward swap rate.

A key objective of this paper is to address the question of whether there is a certain swaption implied volatility quote to be favored. Figure 1 compares the historical time series of implied volatilities for a (10,10) euro at-the-money swaption for both the Black and the Normal quote. Figure 2 plots these volatility quotes against the corresponding interest rate levels. In Figures 1 and 2, the left vertical axes refer to the implied volatility in which a value of 0.5 refers to 50% implied volatility for the Black model and 50 bps for the Normal model.

When considering Figure 1 it is striking that the most recent movements of the Black volatility quotes are massively climbing upwards while the Normal volatility quotes evolve smoothly in the same period. Having in mind the objective of introducing implied volatility quotes, which should allow for a sensible assessment of volatility movements, these diverging evolutions are clearly not desirable. One gets the impression that volatilities are exploding when assessing the Black quote while the Normal quote indicates that the market's view of this risk has not markedly changed.

FIGURE 1: COMPARISON OF BLACK AND NORMAL IMPLIED VOLATILITIES

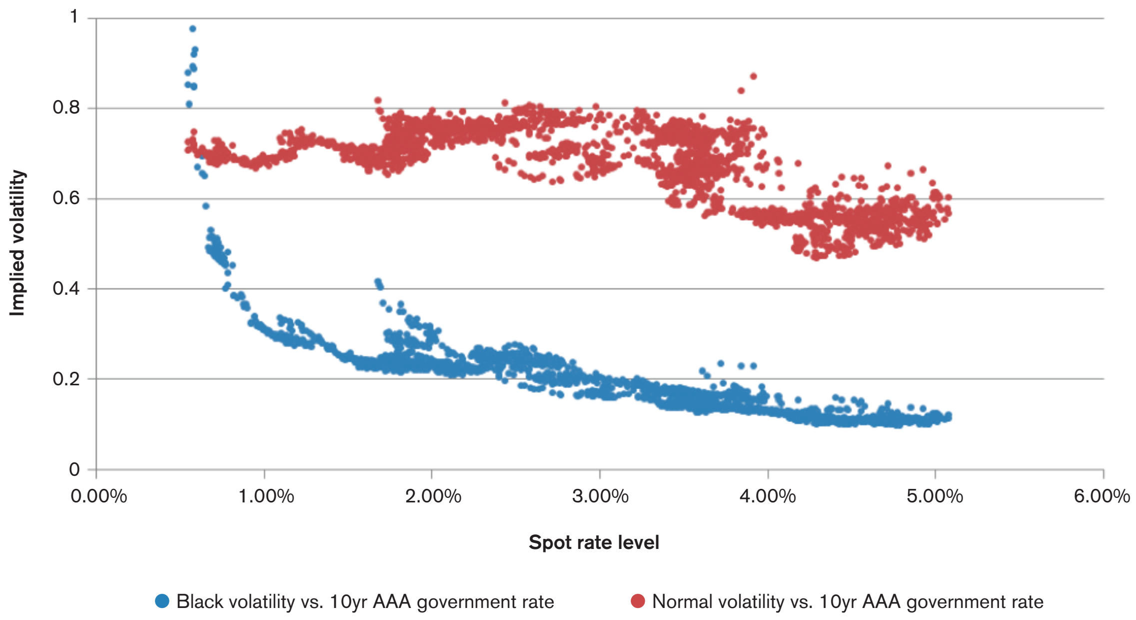


Comparison of Black quote (blue) and Normal quote (red), for historical (10,10) at-the-money euro swaptions and 10-year euro, AAA government rate level (green).

Both the Black and the Normal volatility quote time series have evolved smoothly until Q4 2014, where we see a sudden spike of the Black volatility.

In the same time the underlying AAA government rates have been steeply falling down to about 0.5%.

FIGURE 2: BLACK AND NORMAL VOLATILITIES VS. UNDERLYING INTEREST RATE LEVELS



Comparison of Black quote (blue) and Normal quote (red), for historical (10,10) at-the-money euro swaptions plotted against 10-year euro, AAA government rate level.

While the Normal quote does not exhibit any pronounced dependency on the underlying interest rate level, the Black volatilities tend to increase once the underlying interest rate level is decreasing.

When explaining this phenomenon one needs to turn to the distributional properties of the two underlying benchmark models. Because of its lognormal nature, the Black model is measuring implied volatilities in a relative way, i.e., it measures the volatility σ_{LN} of the relative changes of the forward swap rate. On the contrary, the volatility in the Normal model measures the volatility σ_N of the absolute changes of the forward swap rate.

Considering Figure 2, we can see that over the period of our analysis (2006 to early 2015) the market's view of the absolute volatility of the forward swap rate has not materially changed and exhibits no strong relationship with the level of interest rates, which have fallen significantly over this period. However, the fall in interest rates over the period has impacted relative volatilities, which have increased steadily as rates have fallen and, more recently, very sharply as rates have approached a zero level. This explains the severe movements of the Black volatility while the Normal volatility stays rather calm.

Black volatilities measure the volatility of relative rather than absolute interest rate movements, and are hence linked to the overall level of interest rates.

First conclusions

The steep, declining euro interest rate movements during the latter stages of 2014 and the first half of 2015 have unveiled a major weakness of the Black volatility quote: by measuring the relative volatility (i.e., the volatility of relative rather than absolute interest rate movements), the Black volatility is linked to the overall level of interest rates. While it is not necessarily an issue when interest rates are at a medium to high level, this effect leads to drastic jumps in the Black quotes once interest rates are approaching zero. Therefore, the Black quote fails in its main purpose to separate the effects of what the market perceives as volatility and all remaining factors that impact a swaption price (because the interest rate level has a huge effect), and shows a severe, highly nonlinear dependency on the interest rate level.

ARE INTEREST RATES MORE NORMAL OR LOGNORMAL? SOME EMPIRICAL EVIDENCE

While the previous section implies that a Black quote is accompanied by some severe drawbacks, this section extends the analysis of what might be a 'preferred' volatility quote by considering historical interest rate movements and assessing their behavior (either normal or lognormal). Note that even though the implied volatility itself is not the volatility of interest rate movements, but rather a quote for the market's opinion about this, there is a reasonable expectation of a strong link between these two quantities.[3] Therefore, the question of the distribution of interest rate movements is closely related to the question of whether to use Black or Normal implied volatility quotes for modelling purposes.

The main idea of the following assessments is to measure a 'local' or 'conditional' volatility of absolute interest rate changes based on historical time series data. In other words, rather than deriving an overall empirical volatility of the historical interest rate changes, we measure either:

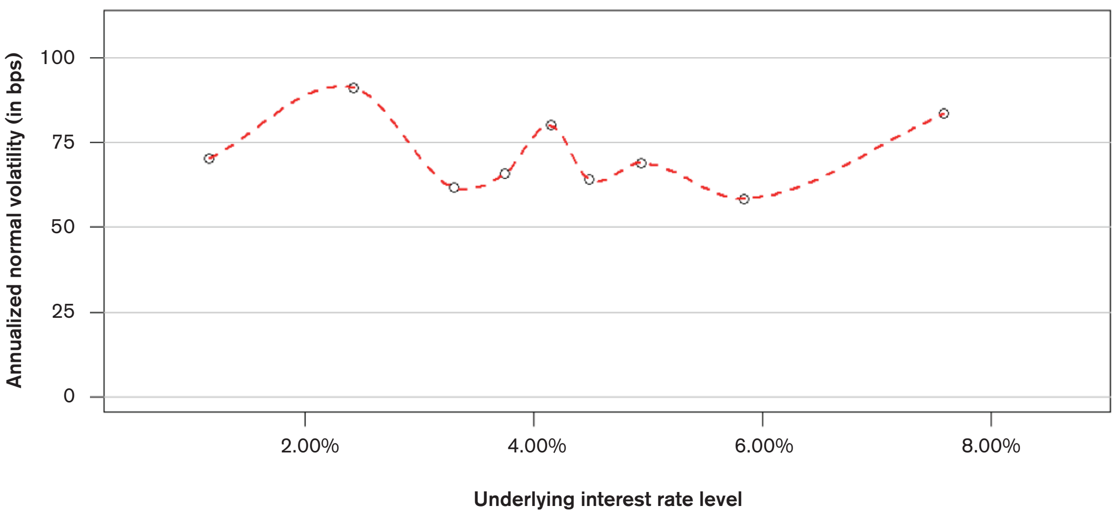
1. The empirical volatility of those interest rate movements that are conditional, based on the fact that the interest rate level has been within a certain range. For this method, we partition the range of historical interest rate levels in subintervals and derive the volatility of the absolute interest rate movements that are conditional based on the interest rate level being in a particular interval. This can be interpreted as a conditional volatility. [5]

2. The empirical volatility of the interest rate movements around any historical point in time. Because volatility can only be inferred by averaging over a certain set of data, and hence, by definition, is not a local quantity, we therefore apply an exponentially weighted average approach to derive the average volatility per historical point in time, averaging over the squared distances of absolute interest changes in a certain time span. This can be interpreted as a local volatility. [1]

In either case, we then plot the conditional and local volatility, respectively, of historical interest rate movements against the underlying interest rate level. A lognormal distribution of historical interest rates would show a positive relationship between the conditional/local volatilities and the level of interest rates. On the other hand, a normal distribution of historical interest rates would show no particular relationship of the conditional/local volatilities and the level of interest rates.

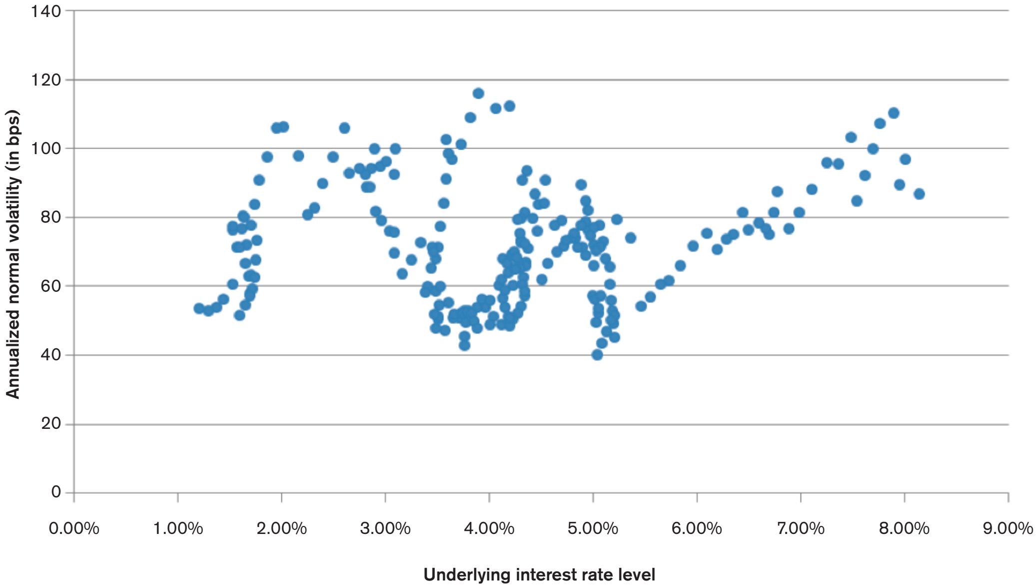
Figures 3 and 4 show the results from both measures above performed for the historical time series of monthly euro 10-year AAA euro government rates from January 1995 to December 2014.

FIGURE 3: CONDITIONAL VOLATILITY



3 Though we accept that implied volatilities are subject to other influences such as the relative demand and supply of derivatives in the market.

FIGURE 4: LOCAL VOLATILITY



Both plots indicate that the level of variability of the interest rates does not depend on the underlying interest rate level.

This is in line with using a Normal model for the modelling of interest rates and the implied volatility quotes for swaptions inferred from this model.

Plot of the conditional (Figure 3) and local (Figure 4) volatility of historical 10-year euro AAA government rate movements against the underlying interest rate level.

Based on these two assessments, we see that higher interest rates have not, at least over the last 20 years, implied a higher volatility of absolute interest rate changes. This analysis therefore provides support for using a Normal model for interest rate movements and swaption quotations.

The authors have carried out similar analyses for government rates of several major currencies and received similar results in all cases.

IMPLICATIONS FOR RISK FACTOR MODELLING

Insurers rely on swaption volatility quotes for the valuation of their liabilities in a market consistent framework as well as for an assessment of their solvency position under the Solvency II framework. This chapter gives an overview on where the drawbacks associated with the use of Black implied volatility quote might affect insurers.

> Keeping Normal volatilities invariant in interest rate stress scenarios typically increases the Black volatility in interest rate down stress scenarios, but decreases the Black volatility in the base scenario (including MA/VA) and interest rate up stress scenarios.

Risk-neutral risk factor modelling

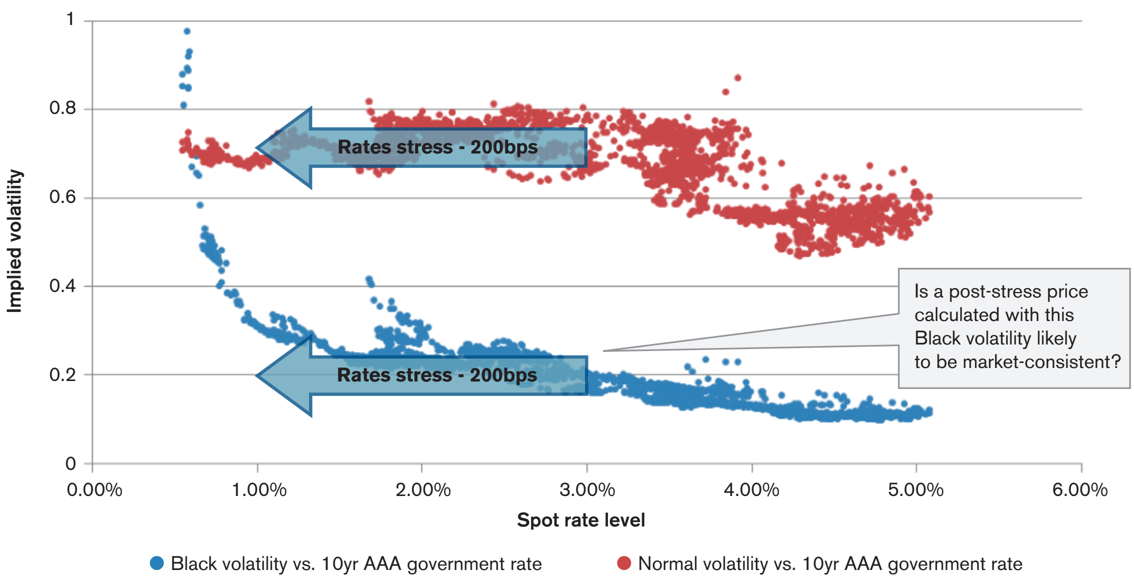
In the market-consistent valuation of their liabilities, insurance companies are projecting their assets under a risk-neutral measure. The essential idea of the volatility calibration in this context is to set the volatility parameters of the interest rate model in such a way that they reflect the prices of selected target swaptions, i.e., that the prices of the selected swaptions inferred via Monte Carlo simulation, based on the underlying interest rate model and its parameters, coincide with certain target prices observable from the market. While this procedure is straightforward in a context where the risk-neutral valuation is performed with the (unadjusted) underlying market yield curve,[4] the following question arises in the presence of risk-neutral scenarios with a nonmarket yield curve, e.g., in an interest rate up or down stress, a valuation including some kind of volatility adjustment (VA) or matching adjustment (MA), or once the yield curve is extrapolated towards some ultimate forward rate.

What does an appropriate 'market-consistent' swaption target look like in a scenario where the underlying interest rate is deliberately deviating from the current market and where there is hence no market to ask for a quote?

In the past, the approach adopted by insurers has been to target implied volatilities rather than prices. To be more precise, the common approach is to keep the implied volatility invariant in interest rate stress scenarios. The rationale behind this approach is that an interest-rate-only stress should not affect any other risk factor. Therefore, the implied volatility quote is used to abstract the price of a swaption from its underlying dependency on the interest rate level. Hence, by keeping the implied volatility invariant in interest rate stress scenarios, insurers can carry over the present market view on volatility to the alternative hypothetical interest rate environment required by a particular stress scenario.

Having in mind the conclusions from the previous section, one needs to make a conscious decision whether to use a Black or a Normal quote for this task. Taking into consideration the relative nature of the Black quote, including its anomalies in the first half of 2015 as well as the historical evidence pointing to a more Normal distribution of interest rate changes, there is in our view a good case for keeping Normal volatilities invariant rather than Black ones.

FIGURE 5: BLACK VS. NORMAL



4 In this case, one would simply ask for the swaption prices as inferred from the Monte Carlo simulation and from the market to coincide.

The table in Figure 6 provides an overview of the implications of these two alternatives for interest rate up and down shifts.

FIGURE 6: IMPLICATIONS FOR INTEREST RATE UP AND DOWN SHIFTS

	IMPLICATIONS FOR INTEREST RATE UP SHIFTS	IMPLICATIONS FOR INTEREST RATE DOWN SHIFTS
Keeping Normal volatilities invariant	Black volatilities are decreasing	Black volatilities are increasing
Keeping Black volatilities invariant	Normal volatilities are increasing	Normal volatilities are decreasing

By keeping Black, i.e., relative, volatilities constant with rising or falling interest rate levels, one is increasing or decreasing Normal, i.e., absolute, volatilities, respectively, while Black volatilities are decreasing or increasing with rising or falling interest rate levels, respectively, once Normal volatilities are held constant.

Insurers that have been following the paradigm of keeping Black volatilities invariant in interest rate stresses will, therefore, when switching to keeping Normal volatilities invariant, implicitly increase the absolute volatility in interest rate down stresses but simultaneously decrease the absolute volatility in interest rate up stresses. Note that the latter class of interest rate stress scenarios does also include what is often referred to as the base scenario[5] once some MA or VA is applied.

Note that the implications discussed in this section might be particularly relevant for the Solvency II standard formula interest rate stresses. These stresses are defined to capture only interest rate risk that arises from changes in the level of the basic risk-free interest rates. Therefore, in order to comply with the fundamental idea of these stresses, insurers deliberately need to decide which paradigm to follow when setting swaption calibration targets in these stresses. The illustrations and discussions in this paper give some strong evidence that keeping Normal volatilities invariant in these stresses makes it easier to demonstrate compliance with the rationale of these two stresses.

Real-world risk factor modelling

Figure 6 above indicates a pronounced negative correlation between the interest rate level and the level of Black volatilities once interest rates are particularly low. The table in Figure 7 illustrates this phenomenon by exhibiting the correlations between the returns[6] of Black and Normal implied volatilities and the returns of the underlying 10-year AAA government rate.

FIGURE 7: CORRELATIONS BETWEEN BLACK AND NORMAL IMPLIED VOLATILIES

	BLACK VOLATILITIES VS. 10-YEAR AAA GOVERNMENT RATE	NORMAL VOLATILITIES VS. 10-YEAR AAA GOVERNMENT RATE
01/2005-03/2015	-54.8%	2.8%
06/2014-03/2015	-75.3%	13.1%

We clearly see a pronounced negative correlation between the returns of the Black volatility and the 10-year AAA government rate over the whole time horizon, plus a very significant increase in this effect over very recent history. At the same time, the correlations between the returns from the Normal volatility and the 10-year AAA government rate are much less significant.

5 I.e. the scenario representing the unstressed best estimate conditions.
6 In this context, the term 'return' refers to the relative change of an underlying variable.

The results shown in Figure 7 support the visual and theoretical findings above, highlighting the complex dependency of the Black quote on the underlying interest rate level. Therefore, insurance companies modelling the real-world distribution of their key market risk factors in the context of internal models are facing the following considerable challenges when modelling implied volatilities via their Black quotes:

1. The marginal distribution of the Black volatilities requires some particularly pronounced tails, allowing for massive up and down movements of the Black volatilities, which are in reality solely caused by the interest rate movements once the overall interest rate level is close to zero.

2. The pronounced and complex, i.e., nonlinear, dependency between the interest rate level and the Black implied volatilities requires a sophisticated copula to allow for a proper joint distribution of the Black volatilities and the corresponding interest rates.

As a consequence, even once the first challenge above is properly implemented, the stand-alone capital for interest rate volatility, which is derived from the 99.5% value-at-risk of the own funds distribution under the real-world risk factor distribution for implied volatilities,[7] can therefore be exceptionally high but will in total be diversified away by the highly negatively correlated capital for interest rates. While this should not distort the total capital requirement, its decomposition into the individual risk factors can appear extremely distorted, sending out the odd message that interest rate volatility is an extremely important risk factor in a univariate setting but basically diversifies away with interest rate risk. Furthermore, this effect can have negative consequences once companies have implemented certain limit systems or risk appetite strategies where intervention thresholds or target risk capital numbers are often defined in terms of individual capital figures per risk factor, which can be confounded by the drawbacks of the Black volatility quote.

The fundamental importance of the second challenge above is revealed once we consider the extremely high Black volatilities as of March 2015. In real-world simulations, implied Black volatilities need to be accompanied, i.e., basically caused by, a significantly low interest rate environment. Otherwise, the resulting real-world scenario does not have a relevant meaning, because any such historical high-volatility event is accompanied by low interest rates. Even worse, the insurers' own funds will suffer a breakdown on real-world paths with medium to high interest rates but exceptionally high volatilities, which are not realistic but an artefact of the Black paradigm. Hence, the occurrence of these paths can have a negative impact on the resulting solvency capital position, which is not justified.

Both issues are automatically addressed once a model with Normal volatilities is applied: because the historical times series of Normal volatilities does not include jumps implied by interest rate levels, its marginal distribution does not display such a pronounced tail. Furthermore, there is strong evidence that the Normal volatility quote itself manages to remove the impact of the underlying interest rate level from the swaption price, which is clearly beneficial for modelling the joint distribution of these two risk factors because this avoids a set-up in which we have to model two risk factors where one is itself a highly significant explanatory variable for the other.

A real-world model for Black volatilities requires some extreme tail for its marginal distribution and a complex copula to join it with the interest rate distribution.

A glance at the banking world

We note that banks face similar challenges in pricing swaptions in the current very low interest rate environment. Whilst we have not undertaken a formal survey of market practice, we understand, anecdotally, that banks have recognised the issues presented by lognormal models and that consequently approaches have moved towards the use of Normal models.

7 Where all other risk factors are assumed to be at their constant base levels.

CONCLUSIONS

We have outlined the motivation for using swaption implied volatilities rather than swaption prices and introduced two reference quotes for this purpose, the Black and the Normal quote. As a next step, we compared the historical time series for both quotes based on a (10,10) euro at-the-money swaption. This showed that the Black quote has tended to be less stable, which traces back to its relative nature, essentially making it dependent on the underlying interest rate level. We then analysed whether the historical AAA government rate data favors the Black or the Normal model, and concluded that the weight of the evidence points to the Normal model reflecting the data more appropriately. In the final section, we presented two applications in the context of risk factor modelling for insurance companies, in which the volatility quote is of enormous relevance. Further, we discussed the potential impacts of this decision on the resulting capital figures. Lastly, we briefly considered how the banks have responded. Overall, we conclude that, in the current extremely low interest rate environment, the use of Normal volatility is a more robust approach.

However, the authors want to emphasise that it is clearly advisable to regularly check whether this statement remains valid at future points in time.

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