Least Squares Monte Carlo for fast and robust capital projections

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INTRODUCTION

Reliable capital projections are necessary for management purposes. For an insurer operating in the Solvency II context, dynamic management actions in the model should reflect the Solvency II coverage ratios in the projection. However, a brute-force calculation of future Solvency II coverage ratios would be numerically burdensome. Having applied Least Squares Monte Carlo (LSMC) to the 1-year value-at-risk calculation in [5], we show in the current article how to extend a one-year application of LSMC to the multi-year setting.

1.1 Motivation

For management of insurance business, it is of utmost importance to understand how future capital requirements and capital coverage evolves under different scenarios. The availability of this information allows the management to react to foreseeable capital shortages by taking appropriate actions.

If the insurance undertaking uses an Internal Model, then it is expected to demonstrate that their Internal Model plays an important role in the undertaking's system of governance, in particular in their risk management process and their decision making process – in the sense of the Use Test as laid out in the Article 120 of the Solvency II directive [1]. With the Solvency II capital coverage set to play an important role in the decision making process, a realistic actuarial projection model for Solvency II purposes should incorporate management actions based on the undertaking's current Solvency II balance sheet numbers in any given projection year. This naturally leads us to the challenge of reasonably estimating Solvency II coverage ratios when assets and liabilities are being projected over time according to a whole set of valuation scenarios (e.g. for MCEV calculations).

Apart from the "Solvency II Pillar I" motivation outlined above, there also are regulatory requirements to provide capital projections in the context of Pillar II. According to the Article 45 of the Solvency II directive [1], *every* insurance undertaking shall conduct its own risk and solvency assessment (ORSA). In particular:

…The own risk and solvency assessment shall be an integral part of the business strategy and shall be taken into account on an ongoing basis in the strategic decisions of the undertaking.

Furthermore, the Consultation Paper 008/2011 [2] provides the following interpretation of the directive:

…The undertaking's assessment of the overall solvency needs should be forward-looking.

…The undertaking needs to project its capital needs over its business planning period. This projection is to be made considering likely changes to the risk profile and business strategy over the projection period and sensitivity to the assumptions used.

In other words, the Solvency II coverage ratios should be assessed over a period of 5 years along some deterministic path(s) considered for business planning purposes.

Both applications require a robust and flexible framework that allows for a projection of the Solvency II coverage ratios over a certain period given some development of the portfolio of assets and liabilities and financial market conditions.

1.2 Calculation of future Solvency II coverage ratios

The task of estimating future Solvency II coverage ratios for a mid-term or long-term projection period is much more daunting than the assessment of Solvency I coverage ratios. Whereas Solvency I coverage ratios can be deduced rather easily from standard balance sheet projection data, calculation of future Solvency II coverage ratios would theoretically require a nested simulation approach.

With the nested simulation approach being extremely demanding in terms of run-times, the alternatives, the standard formula or the so-called *proxy modeling techniques*, such as Replicating Portfolios [3], Curve Fitting [4] or Least Squares Monte Carlo (LSMC) have become quite popular. In our previous paper [5], we showed how to apply LSMC in order to generate a 1-year probability distribution forecast. The series of articles [6], [7] and [8] shows why these approaches can give robust approximations if properly applied, gives confidence intervals for the coefficients of the proxies and an error-estimation in terms of SCR.

1.3 Scope and structure of the paper

In the current paper, we are going to extend the LSMC techniques applied to the 1-year setting in [5] to the multi-year setting. The remainder of our paper is organized as follows:

- In Section 2, we show how to estimate some economic liability variables such as PVFP (Present Value of Future Profits) or BEL (Best Estimate Liability) for an arbitrary projection year t.
- In Section 3, we discuss possibilities to estimate Solvency II required and available capital based on the above.
- In Section 4, we show how the concepts introduced above perform in a realistic German case study.
- In Section 5, we draw conclusions from our results and outline some directions for further research.

2 Estimation of economic liability variables for future projection years

Our goal is to determine the required capital, SCR, and the available capital, also called own funds, of a portfolio of assets and liabilities not only at time 0 but also for future times $t > 0$. The development of the portfolio of assets and liabilities from time 0 to time t as well as the financial market conditions at time t is given by the context:

- When performing stochastic valuation runs e.g. for MCEV purposes these data are a part of the valuation scenarios and their influence on the assets and liabilities when projecting those over time.
- For ORSA purposes, this information is based on assumptions of the business plan. Typically it is assumed that assets earn a best-estimate return and financial conditions evolve smoothly. The business plan thus prescribes one deterministic path for the risk-factors governing the financial condition of the portfolio. It is good practice however to also analyse the capital projection under a small number of different scenarios which are relevant for the portfolio, e.g. a low interest rate scenario, asset shocks etc.

In other words, the challenge is to determine the relevant key indicators, SCR and own funds, for a large number of scenarios and $t>0$. The straightforward solution for this problem would be a nested simulation approach (cf. Figure 1), where the PVFP or the BEL for each scenario and time t is evaluated by using, say, 1000 risk neutral scenarios, each of which calibrated to the scenario and time t. In order to obtain the SCR, one might want to run such evaluations for thousands of different realizations of the risks under a 1-year horizon covering the time step from t to t+1.

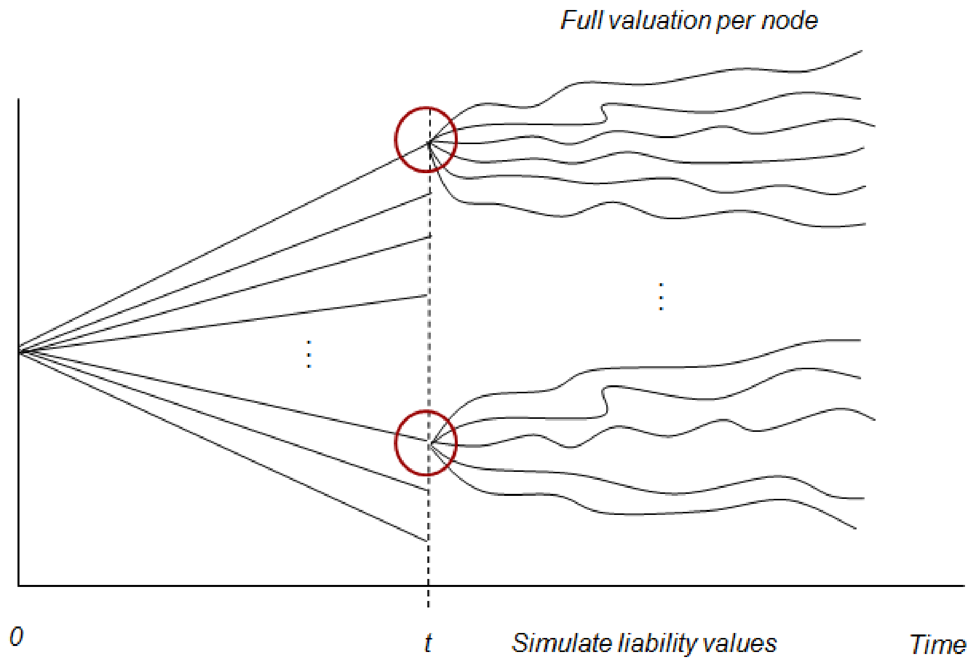

Figure 1: Nested stochastics approach

The risk drivers at time t will not be sufficient to explain the liability value since they do not take into account the values of the risk drivers at T=0,…,t-1. Clearly the historic path of the risk drivers affects the financial situation of the company and thus has an impact on the liability value at time t as well. An insurance company with sufficient buffers, such as unrealised gains, might better sustain a fall in interest rates than a similar company without any such buffers. Thus, in theory we would have to create simulations of all historic paths as well and determine a proxy which approximates the liability value as function of all risk drivers at all times T=0,…,t-1 and at T=t. This does not sound too onerous as the calibration simulations typically evolve from T=0 on and thus the simulations create paths including T=1,..,t-1. But the calibration process becomes very cumbersome as the dimension of the risk driver space becomes very large. Therefore, further explanatory variables, so-called state variables $s_t = (s_{1t}, \dots, s_{lt})$, such as premium income, reserves or unrealised gains must be included in the data used to capture the impact of the history of the risk drivers; their values incorporate the historic path of the risk drivers.

In order to obtain a full functional relationship between the liability value y_t, the risk drivers in t=0 and the state variables we just regard the state variables as risk drivers in time t and redefine the set of risk drivers: $x_t = (r_t, s_t)$. Then we perform an ordinary least squares regression for each t>0 under consideration and get a functional relationship

$$f_t(x_t) = \sum_{j=1}^{k(t)} \beta_{jt}\varphi_j(x_t), \quad t > 0,$$

where $\varphi_j: \mathbb{R}^{h+l} \to \mathbb{R}, j = 1, \dots, k(t)$ form a set of $k(t)$ basis functions. Even though each $y_t^{(j)}$ would be a rough estimation for the corresponding liability value, the regression generates a refined and more accurate liability value where the errors in the $y_t^{(j)}$ are being diversified away as described in [5], [6],[7] and [8] by the least squares estimator.

Having performed the fitting process, we can estimate the liability value at time t for any particular simulation evaluating the liability function f_t for the current values of the risk drivers and state variables of the corresponding simulation.

3 Calculation of Solvency II coverage ratios for future years

Firstly, we have to calculate the required capital (SCR). There are several ways of using the liability function - which has been calibrated using the methodology described in Section 2 - in order to calculate the SCR for a particular scenario and projection year:

I. Assume that the risk drivers $r_{1t},\ldots,r_{ht}$ now, in contrast to the calibration context of Section 2, follow some real world distribution and generate m realisations of these risk drivers. Evaluate the liability function for these m risk driver tuples, where the state variables $s_{1t},\ldots,s_{lt}$ are set to the levels corresponding to the projection scenario considered, in order to adjust the liability function to the proper information on the past and current situation of the company. These m realizations of the liability values are used to derive the full distribution of the liability value for t>0 and the scenario. The corresponding SCR can be calculated as the difference between the liability value at base level and the corresponding 99.5% quantile of the distribution of the liability value as usual. Of course this estimation involves a stochastic estimation error as well.

II. Use a standard formula – style approach. Stress each risk driver individually and evaluate the liability function for this single stress, with all other risk drivers and state variables set to the levels corresponding to the projection scenario considered. This will lead to h individual Solvency Capital Requirements $SCR_{jt}, j = 1, \ldots, h$ at time t for the scenario considered. Aggregate these SCR via a covariance approach based on some correlation assumptions, i.e.

$$SCR_t = \sqrt{\sum_{i,j} \rho_{ij} SCR_{it} SCR_{jt}}$$

where ρ_{ij} is the correlation between the individual risks.

In the subsequent case study, we focus on the standard formula – style approach for the sake of simplicity.

Secondly, we have to calculate the available capital (ASM). We perform our ASM calculations via the following steps:

- Evaluate the current BEL via the corresponding liability function.
- Calculate the risk margin (RM)[1]. For the sake of simplicity, we estimate RM via a cost-of-capital approach based on the individual SCR values for the individual non-hedgeable risks, obtained using the approach II above.
- Calculate the ASM by subtracting BEL and RM from the market value of assets.

4 Case Study

4.1 Setup

In this section we discuss a detailed case study dealing with LRA, a fictitious German life insurer. Its liability portfolio mainly consists of endowments and annuities, but also includes unit-linked and level term contracts. The asset portfolio of LRA mainly consists of bonds (86% of all assets), but also features some property (7%), equity (5%) and cash (2%).

The liabilities include EUR 2 billion in unit-linked reserves and EUR 8 billion in reserves for traditional products, of which EUR 7.7 billion represent endowment and annuity contracts, whereas EUR 0.3 billion of reserves stem from level term contracts. The policyholder bonus reserve called RFB amounts to EUR 0.8 billion, out of which 0.32 billion amount to the so-called free RFB liability buffer:

German background info: RFB is the German policyholder bonus reserve. Part of it – the so-called "tied RFB" – covers the bonus payments officially declared for the following year. Another part of it – the terminal bonus reserve – covers the future terminal bonus payments for contracts which mature

[1] One could have chosen a somewhat different approach to the calculation of Solvency II coverage ratios. Yet whatever the choice be, the theory described above would supply those economic variables required as the ingredients for the methodology chosen.

after the following year. The remainder of the RFB – the so-called "free RFB" – is not tied to any particular payment to a particular policyholder. The free RFB is meant to cover some future bonus payments to policyholders. However, under the German law, the insurer might seek the regulator's approval for usage of parts of free RFB in order to avoid bankruptcy in a financially distressed scenario. Thus, the free RFB is an important buffer protecting shareholders of a German insurer from future capital injections to a certain extent.

The endowment and annuity contracts mentioned above have an average guarantee interest rate of about 3.4%, which is typical for the German market. In the current low interest rate environment, these guarantees are onerous. Moreover, the business model of German life insurance is highly asymmetric in the following sense:

- In "good" scenarios with rather high investment returns, about 90% of these investment returns are passed on to the policyholders according to German regulatory requirements.
- In "bad" scenarios, in which the investment returns do not meet the guarantees, up to 100% of the losses have to be carried by the shareholders, if the insurance company cannot use any buffers anymore to fund the shortfall.

In this context, the LRA reports a dangerously low Solvency II coverage ratio of 78% at projection start calculated using an internal model. This result well reflects the strain which the current interest rates put upon some German insurers in the regulatory context mentioned above, if the insurer – like the LRA – has seen its buffers[2] all but vanish in the challenging market environment in 2008-2012.

If the LRA had had strong buffers at projection start, then it would avoid shareholder capital injections in several bad scenarios to come, yielding a lesser asymmetry in the sense described above, that is to say, a lower TVOG and a higher Solvency II coverage ratio. Without sizeable buffers at projection start, the LRA features a pronounced asymmetry in its business model. This will result in a highly non-linear dependency of LRA's value upon risk drivers such as interest rates, allowing for a challenging case study.

4.2 Methodology and Results

Due to the information given in Section 4.1 we can identify the following risk drivers as relevant for the LRA:

- interest rates,
- equity,
- lapse,
- longevity,
- mortality.

The variable of interest is the PVFP (Present Value of Future Profits). We set a simulation budget of 50'000 scenarios in order to cover a variety of economic situations. For the calibration of the liability function for each year $t > 0$ we choose the 10-year zero coupon bond price and the equity performance at time t as explanatory variables since those are particularly helpful to capture the market conditions. The insurance risks under consideration are lapse risk, mortality risk and longevity risk. These risks are parameterized as multiplicative adjustments applied to the individual base levels. Hence, the lapse risk is expressed by a lapse factor with values between, say, 0.5 and 1.5. A lapse factor of 0.5 signifies a reduction of all lapse rates by the factor 0.5, whereas a lapse factor of 1 refers to the unstressed level of lapse rates. In order to take the buffers of the LRA into account, we consider the free RFB as additional state variable.

Following a standard formula approach for the determination of the SCR by means of the liability function, we use QIS5 shock levels and correlations between the individual risks in order to obtain the aggregated SCR. The aggregated SCR is furthermore adjusted for the operational risk, simply by

[2] An example for an asset buffer would be an equity market value well in excess of the corresponding fund accounting value. An example for a liability buffer would be a high free RFB.

deriving the operational risk per scenario and time step as defined in the QIS5 technical specifications [9].

For our case study, we define the Solvency II coverage ratio as quotient of Available Solvency Margin (ASM) and the overall SCR, where we set the ASM to $PVFP_{base}$ adjusted by the statutory value of the shareholder's equity.

Remark: Needless to say, our approach can be also applied to other definitions of the Solvency II coverage ratio.

Having calibrated a liability function for each year t > 0 based on the data from the calibration run we want to illustrate its application for two different purposes:

- Projecting Solvency II coverage ratios for MCEV type valuation runs.
- Assessing the Solvency II coverage ratio for a set of ORSA planning scenarios.

Before using the calibrated liability functions for the projection of Solvency II coverage ratios we examine the asymptotic behavior of the liability function for different liability functions to get some flavor for their shape and predictive character.

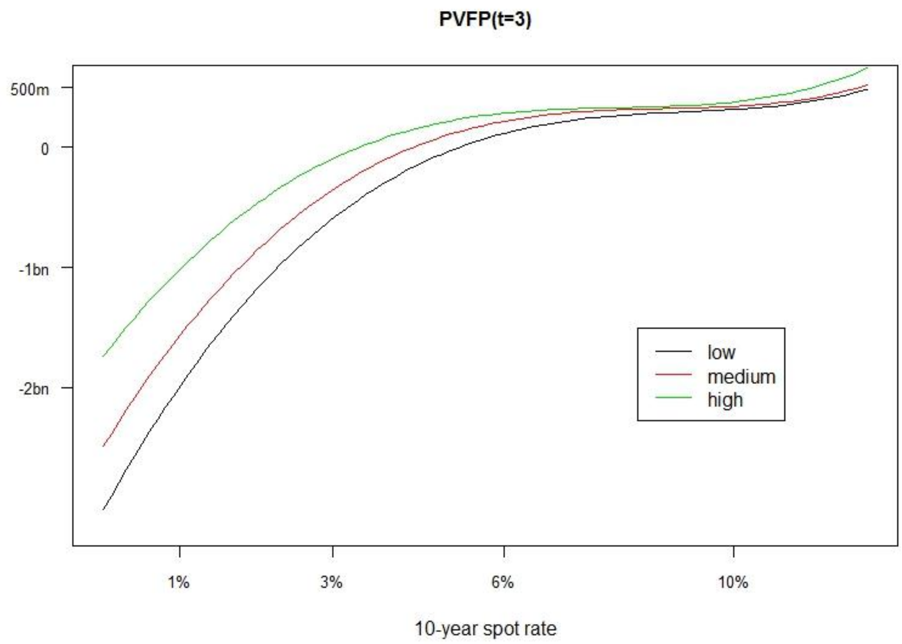

Figure 2 : Plot of PVFP against 10-year spot rate for fixed levels of policyholder bonus reserve at t=3

Figure 2 exhibits the dependency of the PVFP upon the 10-year spot rate at three different fixed levels of free RFB – denoted as "low", "medium" and "high" - please see Section 4.1 for some background info about the notion of free RFB.

If the interest rates are high, then the level of free RFB hardly influences the PVFP. This is because the possibility of a burn-through – which a high free RFB could help to stave off - is remote anyway. However, the free RFB can play a decisive role in times of low interest rates. With a solid free RFB buffer, an insurer could navigate its way through such an environment without running a high risk of shareholder capital injections. In other words: If the interest rates are not sufficient in order to earn the average guarantee rate, then the absence of a buffer leads to losses for shareholders.

Our learning from the above observation is that explanatory variables can replace a full set of history of the "pure" risk drivers such as interest rates or lapses. The free RFB clearly helps "explain" the

PVFP – at least if the interest rates are rather low, the free RFB level has a strong impact upon the PVFP.

Figure 3 shows how the PVFP at t=5 depends on interest rates and lapses. A combination of low interest rates and low lapses leads to an especially low PVFP, since these interest rates are below the average guarantee rate. If the lapses rise with the interest rates kept low, then the PVFP improves – indeed, lapses bring some relief from the guarantee burden. If the interest rates are high and the lapses are low, then the PVFP is high because of interest rate gains. If the lapses rise with the interest rates kept high, then the PVFP decreases – indeed, lapses reduce the base for the interest rate gains.

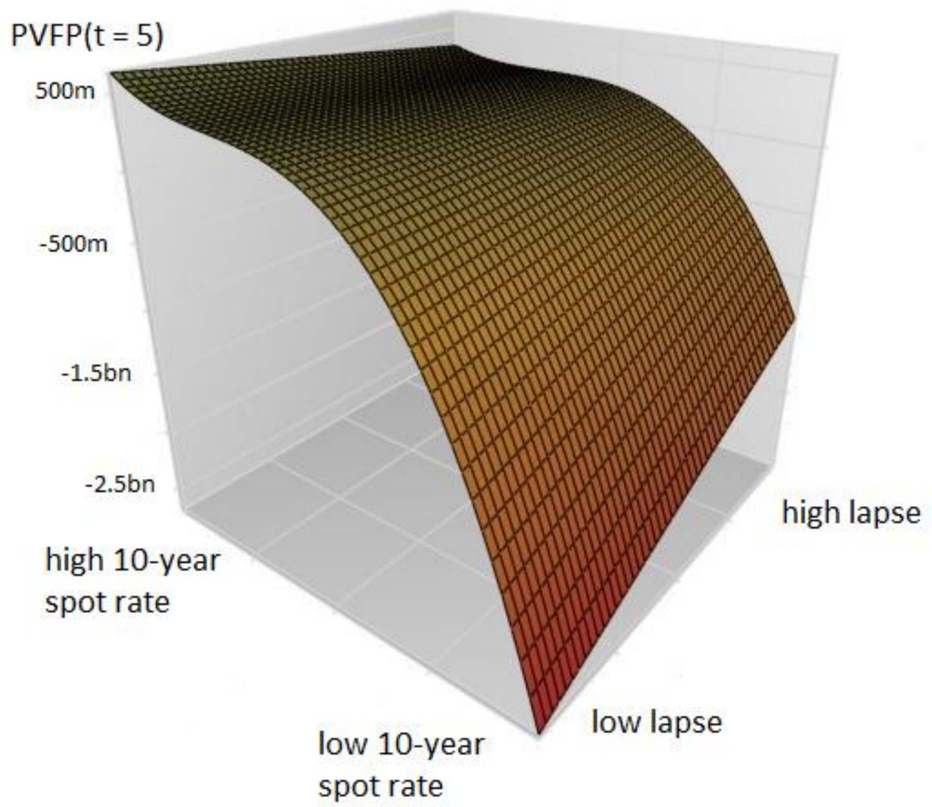

Figure 3: Two-dimensional risk dependency for PVFP at t=5.

4.2.1 Projecting Solvency II coverage ratios for MCEV type valuation runs

We perform two valuation runs and calculate the corresponding Solvency II coverage ratio for each time step from t > 0. For each time step t, we count the number of simulations with

- Solvency II coverage ratio < 33%[3] (red area),
- Solvency II coverage ratio > 33% and < 100% (yellow area),
- Solvency II coverage ratio > 100% (green area).

Here, the first valuation run is performed on the base MCEV scenarios, while the second valuation run is performed in the context of an interest rate down stress. Both valuation runs have been performed with 1000 scenarios each; the tables below display the distribution of the Solvency II coverage ratios in terms of the frequency of red, yellow and green areas for the first 5 years of projection.

[3] Of course other thresholds than those given above could be used as well. In practice, one could infer such thresholds from the regulator's intervention ladder or from a board decision.

Remark: It is beyond the scope of the current paper to evaluate particular Solvency II – driven management actions based on Solvency II coverage ratios. While a wide range of possible actions in such different areas as strategic asset allocation and profit sharing is conceivable, this is a significant challenge in its own right which will be discussed in a separate paper.

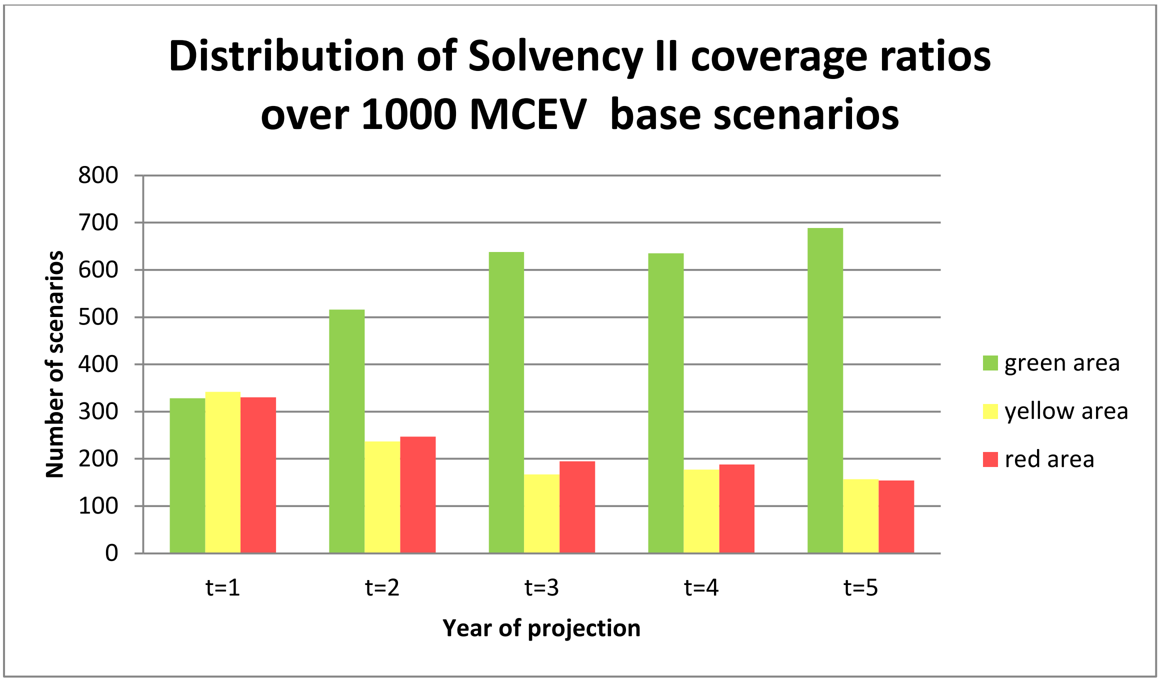

Figure 4: Distribution of Solvency II coverage ratios over 1000 MCEV base scenarios.

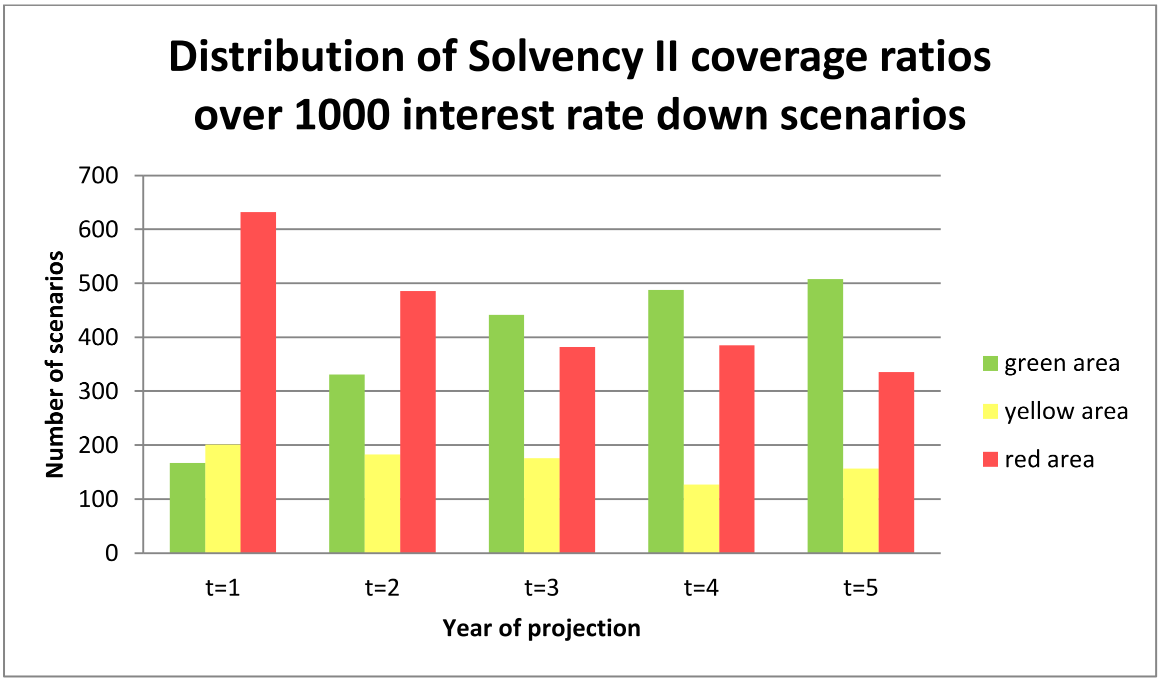

Figure 5: Distribution of Solvency II coverage ratios over 1000 interest rate down scenarios.

Figures 4 and 5 clearly indicate that a downward interest rate shock would significantly decrease the Solvency II coverage ratios. This is an obvious consequence of the LRA's interest rate guarantees, see Section 4.1.

Secondly, we can see the Solvency II coverage ratios improve over time – both in the base valuation and in the stress valuation. This "drift" is partially due to economic reasons such as the gradual change in the portfolio mix – the increasing weight of young sub-portfolios with low interest rate guarantees and the decreasing weight of older sub-portfolios with high interest rate guarantees – and partially due to the closed fund modeling approach.

In order to get some indication for the plausibility of the Solvency II coverage ratios, we consider two exemplary paths from the base scenario set. For these paths, we display the corresponding Solvency II coverage ratios per year (cf. Table 1) and try to explain why these seem appropriate under consideration of the particular characteristics of the corresponding path.

	t = 1	***t = 2***	***t = 3***	***t = 4***	***t = 5***
Simulation 1	62%	103%	156%	34%	79%
Simulation 963	28%	66%	47%	40%	28%

Table 1: Solvency II coverage ratios for two different simulations of the MCEV base scenarios.

<u>Simulation 1</u> (cf. Table 1 and Figure 6)
In the first 3 projection years, we observe a rise of interest rates (in particular, the 1-year rates rise sharply from 1.11% at t=0 to 2.57% at t=3, whereas longer-term interest rates also rise, albeit more moderately), which improves the company's position with respect to the cost of guarantees. Furthermore, we observe a rise of the equity index (from 1.0 to 1.62) and a rise of the property index (from 1.0 to 1.3) over these years. Due to the above developments, the Solvency II coverage ratio improves from 78% at t=0 to 103% at t=2 and 156% at t=3.

Over the next two projection years (t=4 resp. t=5), we see the equity index plummet to 1.09 resp. 0.58. Meanwhile, the property index declines to 1.21 resp. 1.10. The 10-year rates fall from 3.72% at t=3 to 3.43% at t=4, but rise again to 3.76% at t=5. The Solvency II coverage ratio plummets from 156% at t=3 to 34% at t=4, as the company's buffers erode. The ratio improves to 79% at t=5, as the rising interest rates let the TVOG decrease.

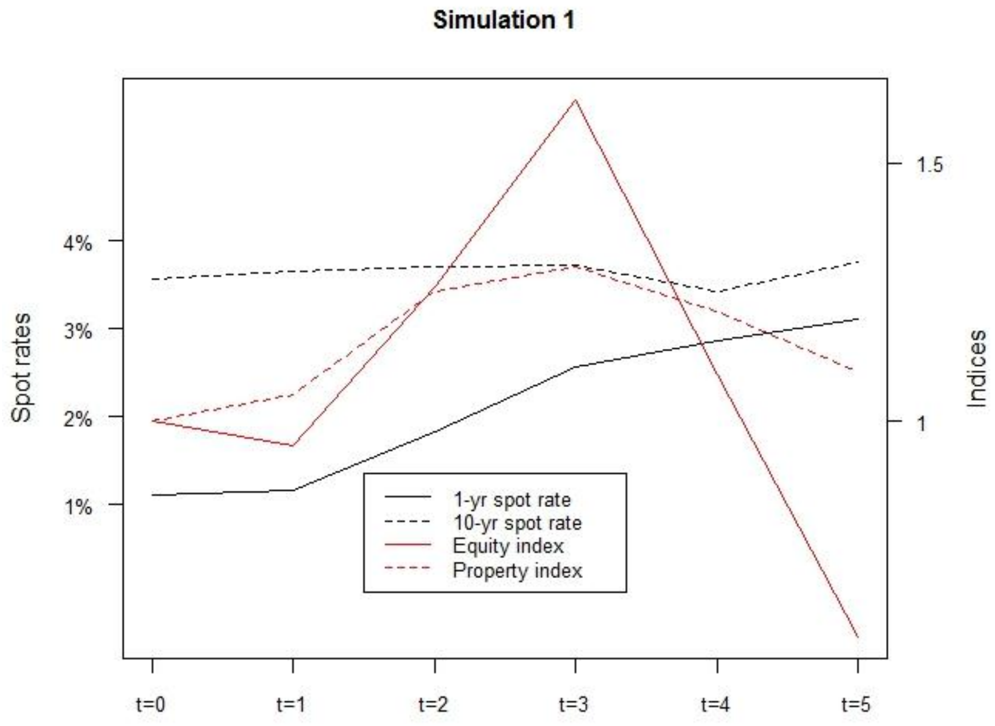

Figure 6: Interest rate yields and indices for simulation 1

<u>Simulation 963</u>(cf. Table 1 and Figure 7)
In the first projection year, the interest rates decline significantly – e.g. the 10-year rate falls from 3.57% to 2.66% - this increases the TVOG and causes a decline of the Solvency II coverage ratio from 78% at t=0 to 28% at t=1. A corresponding upward move of the interest rates in the second year – up to 3.34% for the 10-year rate – triggers a corresponding recovery of the Solvency II coverage ratio to 66%. In the following projection years (t=3 to t=5), the interest rates gradually decline, which causes a steady decline of the Solvency II coverage ratio, all the way to 28% at t=5.

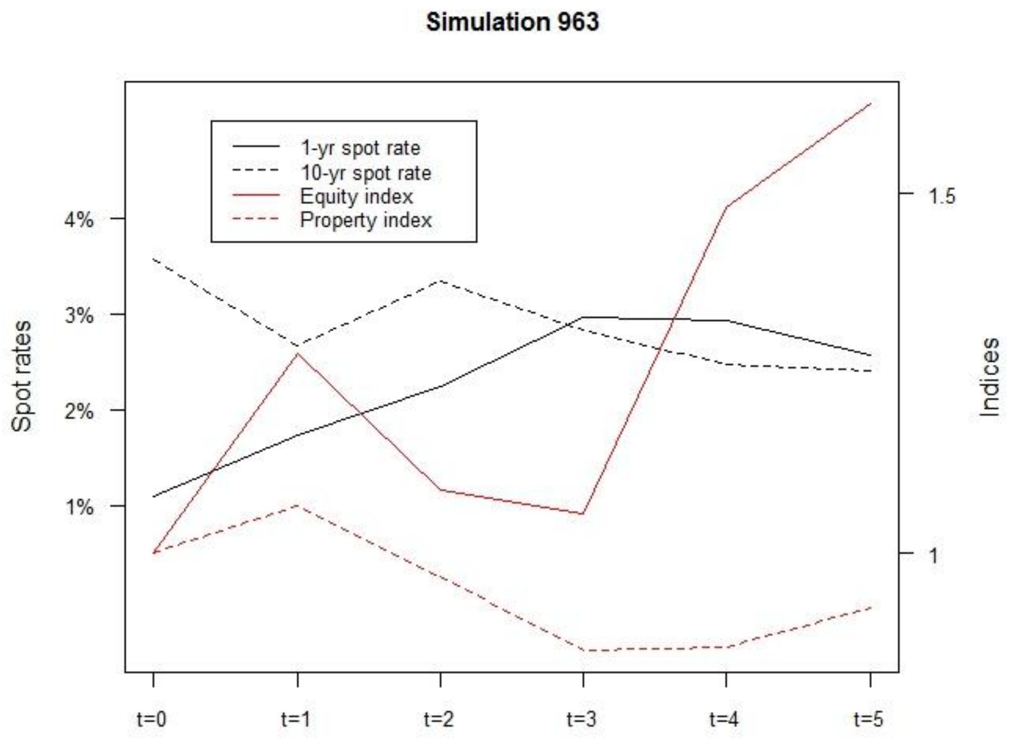

Figure 7: Interest rate yields and indices for simulation 963

4.2.1 Solvency II coverage ratio for a set of ORSA planning scenarios

In order to illustrate how our approach can be used for ORSA purposes, we suppose that the LRA's management has decided to analyze the capital projection under a set of 5 planning scenarios that include constant interest rates for 5 years on three different levels as well as a rise of interest rates after 3 years and a scenario with particular low interest rate level outlook (cf. Figure 8). For the sake of simplicity, the management assumes that interest rate curves are always flat and equity returns exceed the interest rate yield by +200bp.

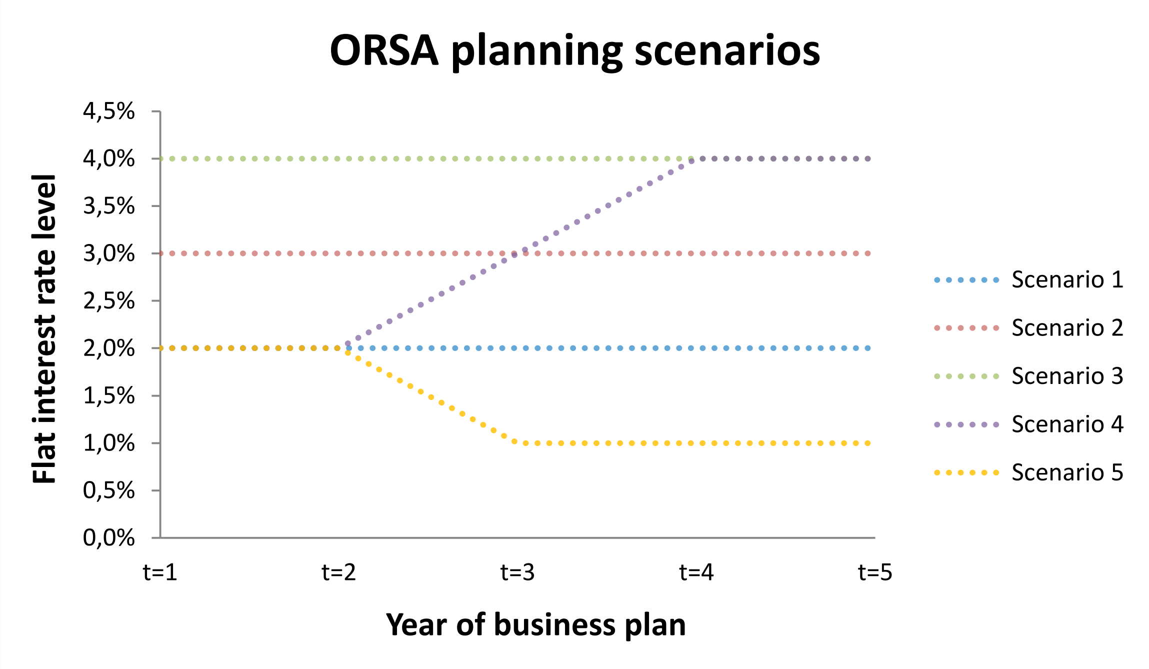

Figure 8: Interest rate yields for 5 different ORSA planning scenarios

	t = 1	t = 2	t = 3	t = 4	t = 5
Scenario 1	-75%	-48%	-16%	0%	-38%
Scenario 2	26%	44%	77%	62%	86%
Scenario 3	93%	96%	100%	119%	147%
Scenario 4	-75%	-48%	82%	170%	140%
Scenario 5	-75%	-48%	-105%	-153%	-227%

Table 2: Solvency II coverage ratios for 5 different ORSA planning scenarios.

The resulting Solvency II coverage ratios for the 5 different ORSA planning scenarios are displayed in Table 2. Scenarios 1 and 5 display a particularly bleak outlook, which is clearly due to the low interest rate yields combined with an average guarantee level of 3.4% at the start of the projection. Note that the negative Solvency II coverage ratios stem from a negative value of the own funds.

Interest rate yields of 3% combined with equity returns of 5% as seen in scenario 2 lead to a subsequent rise of the coverage ratio over the planning period where the yellow areas indicate that the economic situation of the LRA remains tense in this scenario.

Starting with a Solvency II coverage ratio of 78% in t=0 the relatively high interest rate yields of scenario 3 lead to an economic recovery, with the green area reached after 3 years. A similar statement holds for scenario 4 where interest rate yields rise towards 4%.

5 Conclusions

A robust framework for the projection of Solvency II coverage ratios under different kinds of scenarios is a vital tool for insurers:

- It yields input for management rules of actuarial projection systems and helps to comply with Use test demands.
- A forecast of Solvency II coverage ratios throughout a mid-term planning horizon is necessary to manage the business and for ORSA purposes.

Whereas a brute-force nested stochastic approach is numerically burdensome, a proxy modeling solution via Least Squares Monte Carlo is well-feasible.

In the first step, one estimates the values of an economic balance sheet variable such as PVFP or BEL for every scenario and projection year within the planning horizon. This ground-laying economic variable is expressed as a function of several explanatory variables. The latter do not only include all the relevant market and actuarial risk drivers at time T as would have been sufficient for a 1-year VaR calculation. Some additional state variables are necessary in our multi-year application. Indeed, in order to evaluate the Solvency II position of the insurer in a projection year t>1 of a particular scenario, one has to take the "history" [0, t-1] of the insurer in that scenario into account. This is approximated by adding some state variables, e.g. those which carry information about the company's buffers.

In the second step, the Solvency II results can be estimated from the results of the first step. For the sake of simplicity, one may calculate the individual SCRs for each risk driver and aggregate these using a correlation approach. Of course, more sophisticated approaches are also possible.

We have seen how these approaches apply to a realistic German case study. There is no "absolute truth" as to which is the theoretically correct Solvency II coverage ratio for an insurer in a particular scenario and a particular projection year – such numbers depend on a number of important assumptions and are thus prone to some uncertainty. However, we have shown that an extension of a well-established 1-year application of LSMC to the multi-year setting provides a powerful vehicle for Solvency II forecasts.

6 References

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