Milliman Research Report

Prepared by:

Manuel Tschupp, MSc ETH

December 2011

Application of Credibility Theory to Group Life Pricing

Extended Techniques

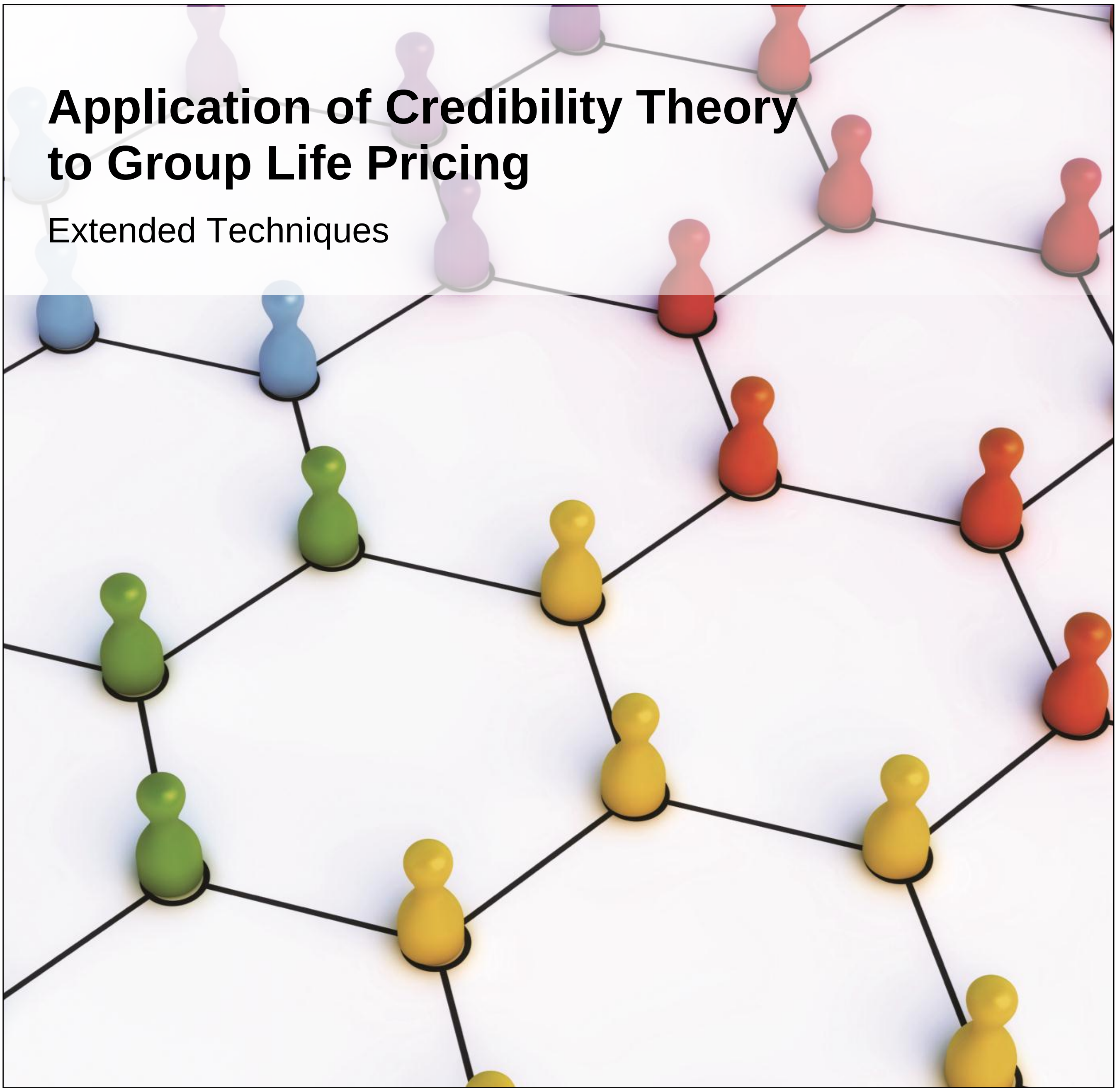


TABLE OF CONTENTS

1.	**Introduction**	**3**
	1.1 Motivation	3
	1.2 Fundamentals	3
	1.3 Structure	4
2.	**Combination of risk measures**	**6**
	2.1 Motivation	6
	2.2 Multidimensional credibility to combine risk measures	8
	2.3 The formalised non-differentiated risk premium	8
	2.4 Serial constitution of the risk-differentiation	10
	2.5 The resulting credibility model	10
3.	**Continuous evolution of risk judgments**	**13**
	3.1 Motivation	13
	3.2 Multidimensional credibility to model time dependence	14
	3.3 Constitution of the structural parameter matrices	15
4.	**Poor underwriting quality**	**17**
	4.1 Motivation	17
	4.2 Adjustment of the credibility weight	18
5.	**Permanent change in risk characteristics**	**22**
	5.1 Motivation	22
	5.2 Amendment of the parameter estimators	22
6.	**Further issues**	**25**
	6.1 Inclusion of external risk information	25
	6.2 Limited individual pricing	26
	6.3 Separate treatment of large contracts	28
	6.4 Outlier observations	30
7.	**Some final thoughts**	**31**
Appendix		**32**

1. Introduction

1.1 Motivation

Credibility theory is a branch of Bayesian statistics and can be used to handle risk-adjusted pricing in group life insurance. The most widely-used credibility model of Bühlmann and Straub provides an estimator that merges a *global* and an *individual* risk experience within a certain observation period in a linear combination, giving each component the corresponding statistical significance as weight.

In August 2011 Milliman published a research report, [T11], on the 'Application of Credibility Theory to Group Life Pricing' based on [T08]. That introductory paper highlighted the basic concepts of how to determine risk-adjusted premiums in group life business for disability and mortality risks. A special focus was given to the application of the credibility model in such a way that non-homogeneous effects in time and common properties of group life insurance business are taken into account. The following Subsection 1.2 outlines the notation used, and the key results.

However, there are more issues an insurer might encounter in the course of daily business, and they might not be addressable by the methodology presented in the aforementioned report. In fact, all of the topics presented here are motivated by practical problems encountered from actual insurance practice. This paper is concerned about extended techniques to cope with various aspects of business reality in different countries around the world. The prerequisite is the same as in the preliminary report, and therefore, the interested reader is referred to [T11] to become familiar with the general background. One might also consult [BG05] to learn about credibility theory in a more general facet.

Lastly, it should be emphasized that the techniques presented in both the introductory and the current report are not only applicable to group life business. Rather, the ideas are generic, although if applied to other types of business, e.g., banking, health, or non-life insurance, careful consideration should be given to the appropriateness of the application.

1.2 Fundamentals

As described in the preliminary report, [T11], we presuppose the existence of a *non-differentiated risk premium*, Π, that covers the best estimate *average* expected total claims amount of a group life contract for a certain period, plus some risk margin to address the inherent uncertainty within. This premium may reflect, for example, the age and gender of each life, and their chosen insured risks, but not however the risk characteristics of the specific group of insured lives as a whole. Due to adverse selection one has to apply a risk adjustment, $\xi > 0$, which then results in the *differentiated risk premium*, $\pi = \xi \cdot \Pi$.

Thereafter, in practice, pricing is then realised within a two-layer approach. To allow for risk differentiation of small volume contracts, and to handle new business contracts where there is no underwriting information available, the portfolio is divided into different risk groups. Each of those risk groups is allotted a risk level ϱ, representing the risk level of an average contract within the risk group with respect to the risk level of an average contract in the whole portfolio. Further, a specific contract is assigned the risk level γ in relation to the corresponding risk group. The actual risk adjustment of this contract is then determined as the product of both components, i.e. $\xi = \varrho \cdot \gamma$.

The generic credibility model is used to provide those risk levels ϱ and γ. It incorporates an *individual entity* i, with the individual risk experience R_i, and a *global entity*, with the global risk experience $R = 1$. The resulting credibility estimator,

$$\varphi_i = \alpha_i \cdot R_i + (1 - \alpha_i) \cdot R \,,$$

is a relative judgment of the risk characteristics of the individual entity with respect to the global entity. The quantities used are

$$R_i = \frac{\sum_{j \in T} C_{ij}}{\sum_{j \in T} f_j \cdot V_{ij}} \quad \text{and} \quad \alpha_i = \frac{\sum_{j \in T} f_j \cdot V_{ij}}{\sum_{j \in T} f_j \cdot V_{ij} + \frac{\sigma^2}{\tau^2}},$$

where C_{ij} represents the *claims observation* (e.g., the number of claims, or the total claims amount), and V_{ij} the *volume* (e.g., the number of insured lives, or the non-differentiated risk premium), of a given individual entity i within the year j of the observation period T.

f_j, σ^2 and τ^2 are *structural parameters* and are considered to be constants for the period of recurring pricing. They are estimated on the basis of the insurer's business in force and are also applied to new business. The expected claims observation frequencies f_j allow for non-homogeneous effects in time, which is often not taken into account when classical credibility models are applied.

To expand beyond the scope of the preliminary report, [T11], this credibility model can be interpreted as a *one-dimensional standardised frequency model* based on Bühlmann and Straub. There, the estimation of the structural parameters σ^2 and τ^2 was not addressed formerly. We provide the general formulae here, and some of the following sections will discuss modifications of the credibility model that affect the estimation of the structural parameters:

$$\sigma^2 = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{n-1} \sum_{j=1}^{n} w_{ij} \left(R_{ij} - R_i\right)^2 \quad \text{and} \quad \tau^2 = \max\left[0\,,\, c \cdot \left\{\frac{I}{I-1} \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} (R_i - \bar{R})^2 - \frac{I \cdot \sigma^2}{w_{**}}\right\}\right],$$

where

$$c = \frac{I-1}{I} \left\{\sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} \left(1 - \frac{w_{i*}}{w_{**}}\right)\right\}^{-1}, \quad \bar{R} = \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} R_i\,, \quad w_{i*} = \sum_{j=1}^{n} w_{ij}\,, \quad w_{**} = \sum_{i=1}^{I} \sum_{j=1}^{n} w_{ij}\,.$$

Here, I represents the *number of individual entities* i that build the global entity (e.g., the number of contracts in a risk group, or the number of risk groups in the insurer's whole portfolio), and $n = |T|$ is the *number of years* within the observation period. The weight $w_{ij} = f_j \cdot V_{ij}$, the 'observation-related volume', can be interpreted as the expected claims observation of the individual entity i within the year j, at the given point of time. Finally, $R_{ij} = C_{ij}/w_{ij}$ is in individual relative observation.

Furthermore, extended estimators for such parameters (and matrices) will be presented in sections where we introduce *multi-dimensional* credibility models.

1.3 Structure

Rather than being an integral piece of work, this document is structured as a collection of sections where each section addresses a certain topic and presents a suggestion on a possible solution to the issue. Nevertheless, the reader will find connections and similarities between the sections. Below is a brief overview about their content:

Section 2 provides a suggestion on how to combine the information from different claims observations measures, such as claims numbers, and claims amounts, to result in one single risk judgment. We present a possible way to take both, or more, sources of information *simultaneously* into account. Thus the possible (and probable) correlation between risk measures is respected. Hence, the reader is introduced to a *multidimensional* credibility model.

Section 3 addresses the problem of unwanted jumps in the risk premium of a given contract from one year to the next. Generally, a risk differentiation system relies on a certain observation period. Once an exceptional risk observation passes out of this period, the risk adjustment might be heavily affected due to the linearity of the credibility estimator. Consequently, insurers usually have to intervene manually, or introduce some artificial dampening mechanism to avoid such effects. This section discusses a special application of multidimensional credibility theory to provide a sound and natural solution to the issue.

Section 4 presents a suggestion to handle poor underwriting quality of a given contract or sub-portfolio. This issue might typically arise with new business, when an insurer has to rely on external underwriting data to determine a risk-adjusted premium. There might also be a subset of contracts in the insurer's whole portfolio where increased uncertainty in the risk observations is present, e.g., in microinsurance markets. If such an uncertainty itself, or the extent to which it might affect the weight of the individual risk experience in the risk judgment, is quantifiable, then this section demonstrates the impact on the risk premium.

Section 5 proposes a preliminary stage to an evolutionary credibility model. Generally, risk differentiation systems assume the risk characteristics of a given contract to remain constant within the observation period. However, such an assumption might be unrealistic in many real-world business situations. For example, changes in personnel of a company affect the risk characteristics of the considered group of insured lives, and thus the characteristics in an observation period should rather be considered as realisations of a stochastic process. The approach presented in this section respects that fact by including a non-diversifiable variance component (i.e. some type of risk margin).

Section 6 considers further issues and possible solutions. First, the question of how to incorporate external risk information about a group of insured lives is addressed. Second, in some countries or for some insurers there might be restrictions on the incorporation of individual risk experience below a certain volume of a contract. There, a very simple approach is suggested to adjust the model. Third, insurers might aim to evaluate the risk characteristics of large volume contracts more independently from the residual portfolio, as for some reason different risk behaviour is suspected within large companies. Lastly, the treatment of outlier observations within the observation period is addressed. Extraordinary large claims might affect the risk judgment to a considerable extent, though they might not exhibit much statistical significance in terms of the risk characteristics.

Section 7 leaves the reader with some final thoughts about the topics presented, as well as a preview of evolutionary credibility theory. An evolutionary model allows for a steady change in risk characteristics over time. Hence, the independence assumptions of the Bühlmann and Straub model are no longer appropriate. Although an evolutionary credibility approach seems to more naturally represent business reality, it leads to a more complicated estimation of model parameters and an increased need for statistical data. Therefore, applicability might sometimes be questionable for some insurers or some types of risk. This section introduces the basic idea and reveals some of the possible pitfalls.

2. Combination of risk measures

This section concerns the estimation of the risk adjustment for a group life insurance contract, taking both the observed number of claims and the observed claims amounts into account. To respect the possible correlation between the two measures, the reader is first introduced to a multidimensional credibility model. In addition, the issue of unwanted correlation between the non-differentiated and the differentiated risk premium is addressed, to avoid double consideration of certain risk criteria.

2.1 Motivation

In Subsection 1.2, we revisited the fact that the claims observations C_{ij} used in a pricing regime can be defined in different ways. Typically, in insurance practice it is either taken to be the observed number of claims or the known total claims amount. In [T11], Subsection 3.4, it was suggested to apply the technique presented to either of those measures, and to combine the two resulting risk adjustments into one single judgment incorporating both sources of information. Or, alternatively, the measures could be merged to apply the credibility model just once.

Both methodologies demonstrate a certain limitation of the regime under consideration. The reason is that the proposed credibility model is *one-dimensional*. As we have two (or more) different claims measures, we seek to compute the one-dimensional credibility estimator separately, and thus at some point have to consolidate the information in some way. However, what is the right approach to do so if the observed number of claims and the average claims amount of such a claim are (positively or negatively) correlated? The technique might then result in a double count, or the two corresponding risk-judgments could inappropriately outweigh each other.

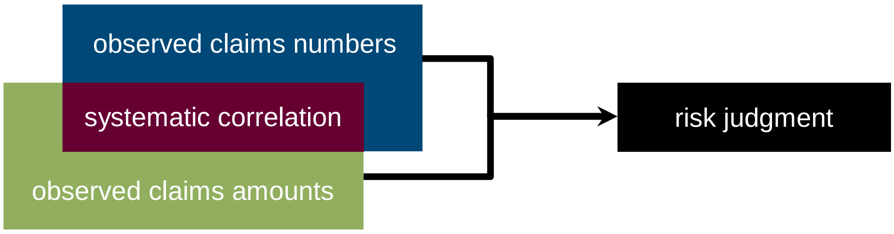


Figure 1: Combination of risk measures

In terms of group life insurance, there are examples observable in actual practice where the claims frequency and claims severity are clearly correlated:

- Some insurance markets may incorporate different *waiting periods* for insurance benefits for work incapacity and disability risks. Insurance benefit is paid after the initial waiting period has passed. As the claims amount includes the expected future benefits, which is affected by the high recovery probability in the waiting period, the corresponding claims amount can be small. The claims that last longer, perhaps after a second waiting period and further examination, are the fewer, but more severe ones.

- A large company that employs many women within a certain age range might be likely to observe an accumulation of claims of work incapacity due to pregnancy. Those issues are usually temporary, hence a quick recovery is usual and the final claims amount is low.

- From a statistical point of view, there are relatively many deaths and/or disability claims in the construction industry, but the average claims amount is small due to relatively low salary and low insurance coverage. In contrast, in industry sectors with highly specialised work (e.g. research, chemistry, or finance), or for Board-level employees across most industries, the claims ratio is low, but a single claim might be high.

- In periods of financial downturn, evidence suggests that disability claims typically increase in number. An insurer might observe, that different groups of insured lives in the portfolio are affected to different extents, i.e. the influence is non-homogeneous.

Multidimensional credibility theory is concerned with the estimation of several components of a vector (rather than a single value) *simultaneously*, taking their correlation into account. However, using the one-dimensional estimators from [T11] as components, that incorporate the total number of claims and the total claims amount as measures, would result in a systematically very large correlation simply due to construction. Thus, estimation of the portion of correlation that is of interest can be difficult.

Moreover, in [T11] the reader was left with a further issue arising from the construction of the differentiated risk premium. As described in Subsection 1.2, we assume the final risk premium to be a product of the *non-differentiated risk premium* and the *risk adjustment*. Despite the advantages of such a multiplicative approach, one might encounter the following situation:

- Assume the *non-differentiated risk premium* of a group life insurance contract is the sum of the best estimate expected claim amounts of the insured lives, plus some risk margin. Such an expectation might incorporate the age, the gender, the salary, as well as the product-specific insured coverage (and maybe other parameters) of each insured life.

- Further, the *risk adjustment* reflects the risk characteristics of the group of insured lives as a whole. It is usually based on experience rating techniques and incorporates the observed claims of a given contract within a certain observation period.

- One might easily observe that generally, the disability and mortality probabilities of a given insured are dependent on its age, gender, and even salary. Thus those probabilities, as outlined above, might have a major impact on the non-differentiated risk premium, and at the same time influence the number and amounts of observed claims within the group of lives under consideration.

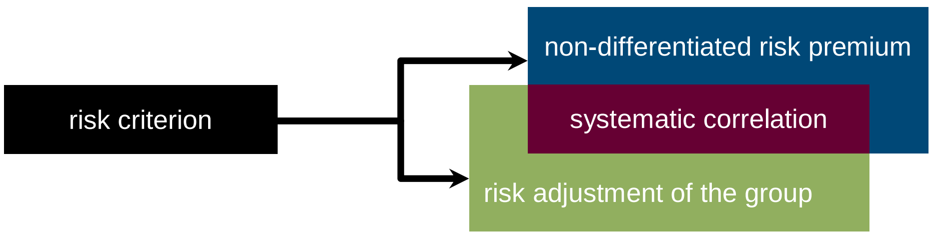


Figure 2: Correlation of the premium components

Therefore, due to the shared risk criteria of the non-differentiated risk premium and the risk adjustment, those two components of the differentiated risk premium might be *correlated*. And hence, this might result in *double penalty*, or *double reward*, or an *inappropriate outweighing* of the same risk criteria, depending on the actual construction of the non-differentiated risk premium. This is illustrated in Figure 2 and should clearly be avoided where possible.

In the following subsections, we suggest approaches to incorporate all of the previously mentioned aspects within the one-dimensional credibility model, but use multidimensional credibility theory and modified risk measures to also address correlation issues, and to combine both claims frequency and severity in a single risk judgment.

2.2 Multidimensional credibility to combine risk measures

In a multidimensional credibility model, the credibility estimator is a vector. Similar to the one-dimensional case described in Subsection 1.2, our aim is to determine the risk adjustment ξ_i of contract i to compute the differentiated risk premium $\pi_i = \xi_i \cdot \Pi_i$. To do so, we compute one credibility estimator based on each of the frequency and the severity of the observed claims. The resulting credibility vector is then

$$\vec{\varphi}_i = (\varphi_i^{\mathrm{n}}, \varphi_i^{\mathrm{s}})' \,,$$

where φ_i^{n} is the (one-dimensional) credibility estimator based on the observed number of claims, and φ_i^{s} is based on the observed claims amounts. Within the multidimensional credibility approach, the estimation of the two components occurs *simultaneously*, taking their correlation into account.

As discussed in Subsection 1.2, it is useful to consider a two-layer approach.[1] That is, a first credibility vector, $\vec{\varrho}_{g(i)} = \left(\varrho_{g(i)}^{\mathrm{n}}, \varrho_{g(i)}^{\mathrm{s}}\right)'$, contains relative judgments of the risk group $g(i)$ in relation to the insurer's whole portfolio, and a second credibility vector, $\vec{\gamma}_i = (\gamma_i^{\mathrm{n}}, \gamma_i^{\mathrm{s}})'$, estimates the risk levels of contract i in relation to the corresponding risk group $g(i)$.[2] One could then consider the vector of risk levels of contract i in relation to the insurer's whole portfolio,

$$\vec{\xi}_i = (\xi_i^{\mathrm{n}}, \xi_i^{\mathrm{s}})' = \left(\varrho_{g(i)}^{\mathrm{n}} \cdot \gamma_i^{\mathrm{n}} \,,\, \varrho_{g(i)}^{\mathrm{s}} \cdot \gamma_i^{\mathrm{s}}\right)'.$$

To achieve a single risk adjustment ξ_i, rather than a vector, one might conceivably propose multiplying the components of the vector $\vec{\xi}_i$, as both ξ_i^{n} and ξ_i^{s} are relative judgments. However, if the former is defined upon the number of claims, and the latter relates to the *total* claims amount, which is obviously also directly affected by the number of claims, then there would be a quadratic impact on the product (or, as a different statement, nearly full correlation would have to be taken into account by the model). Thus, the components should be defined in a different way. This aspect is considered further in the following subsections.

2.3 The formalised non-differentiated risk premium

Subsection 2.1 described the possible systematic correlation between the non-differentiated risk premium and the risk adjustment. Their commonalities need to be formalised in order to modify the risk measures for the credibility model in an appropriate manner.

First, consider a single insurance benefit (e.g. one of several insurance benefits of an insured live in a group life contract) for a given period. It is a widely accepted methodology to compute the corresponding risk premium[3] as the product of the following parameters:

[1] The interested reader is referred to [T11] for a thorough discussion on the two-layer approach.

[2] The function $g: \mathbb{N} \to \{1, \dots, n\}$ maps the index of a contract to the index of the corresponding risk group.

[3] Risk premiums are often computed with reference to an equivalence principle. This states that the expected realised insurance benefit to be received by the insured is equal to the risk premium to be paid by the insured, with allowance for some statistical deviation by an additional margin.

- p represents the **a priori claim probability** of the insurance benefit under the given conditions, i.e. the likelihood that the insured event will be realized within the insurance period.

- b stands for the expected **present value** of the insurance benefit, i.e. the current representation of all future payments as a single value, given that the insured event has been realized.

For the disability and mortality risks in group life business, and pricing on a yearly basis, the a priori claim probability is related to an individual insured live in a group life contract. It indicates the likelihood of temporary working incapability, further long-term disability, or death, during the year under consideration. The possible benefits and expected future development of the claim are captured by the corresponding expected present values.

The *a priori* knowledge refers to the fact, that an insurer does not include experience rating of the contract under consideration when computing the aforementioned premium. Usually, there are probability tables differentiated by insured risk, age, gender, and/or other a priori known criteria, of an insured live. Such tables might be estimated on the basis of the insurer's whole portfolio, and they might further incorporate external information such as shared statistics amongst several insurance companies.

In the described situation, the non-differentiated risk premium, Π_{ij}, of an individual entity i (e.g., a contract) during the year j is the total of the aforementioned risk premiums Π for all insured benefits of all insured lives within the individual entity. An insurer might now observe that, for the disability as well as the mortality risk, the claim probability, p, generally increases with the age of the insured life. At the same time, the present value of the insurance benefit, b, might typically decrease with the age of the insured life. Their product demonstrates the expected behaviour of the risk premium for the insured event, which consequently also impacts Π_{ij}. These relations are qualitatively illustrated in Figure 3:

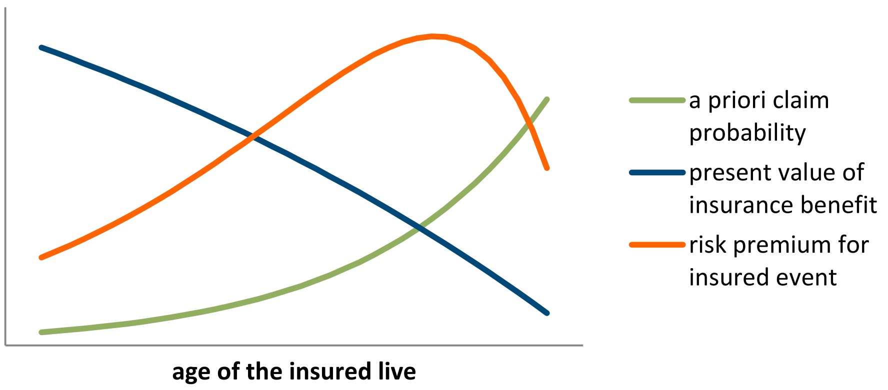


Figure 3: Qualitative construction of the non-differentiated risk premium for temporary annuities

It is a natural property of the *a priori* claim probability to also influence the actual *a posteriori* claims observation of a group of insured lives, though the latter might be further affected by the risk characteristics of the group. Moreover, the average present value of insurance benefits within the group of insured lives certainly affects the average observed claims amount. Therefore, the non-differentiated risk premium Π_{ij} and a risk adjustment ξ_i, which is directly based on observed claim numbers and severities, are heavily correlated by construction.

2.4 Serial constitution of the risk-differentiation

The suggested solution to this issue is to base the first standardised relative risk measure of the credibility model on the a priori information within the non-differentiated risk premium, such that the deviation from the a priori expectation is measured. The second standardised relative risk measure shall consecutively be based on the first measure, as well as the non-differentiated risk premium, to quantify the deviation in the risk observation, conditional on the given previous observations.

This can be thought of as a serial constitution of the risk-differentiation system. As a consequence, the non-differentiated risk premium, Π_{ij}, the risk adjustment based on the frequency of observed claims, ξ_i^{n}, and the risk adjustment based on the severity of observed claims, ξ_i^s, can be multiplied without causing systematic correlation by construction.

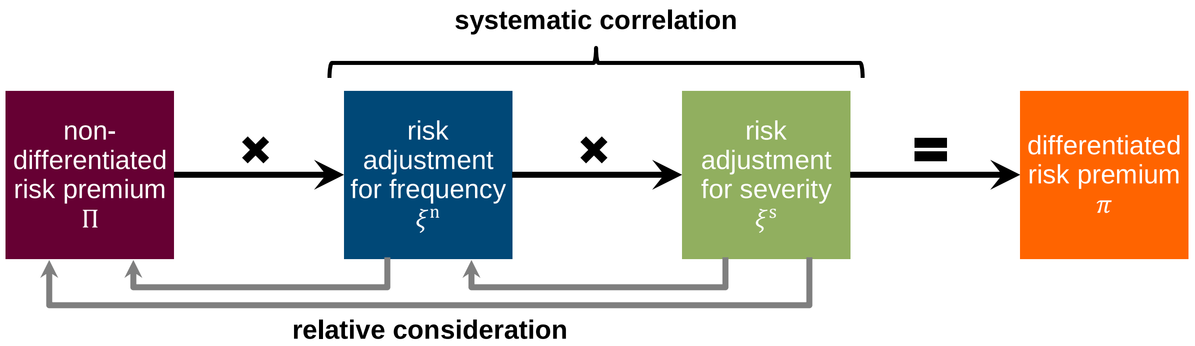


Figure 4: Serial constitution of the risk-differentiation

A formal construction of the principle is discussed in the following subsection.

2.5 The resulting credibility model

In order to be in line with what was presented in [T11], we rely on the multidimensional credibility model of Bühlmann and Straub. Consider the underwriting data

- N_{ij}, the **observed number of claims** of the individual entity i, that have arisen during the year j within the observation period T, and are currently known, and
- S_{ij}, the **observed total claims amount** of the individual entity i, that has arisen during the year $j \in T$, and is known at the same point of time, where
- L_{ij} is the **number of insured lives** of the individual entity i during the year $j \in T$.

Let further the individual entity be the consolidation of all insured lives $k \in \{1, \dots, L_{ij}\}$, and the global entity the consolidation of all individual entities $i \in \{1, \dots, I\}$. Moreover, there is an a priori claim probability per insured live, i.e.

$p_{ij}^{(k)}$ represents the a priori claim probability of the k'th insured live within the individual entity i during the year j.

Firstly, the observed number of claims, N_{ij}, should be compared to the a priori expected number of claims within the individual entity. This a priori expectation, in line with the considerations from Subsections 2.3 and 2.4, can be computed by the total a priori claim probability of the individual entity by an independence argument, i.e. $p_{ij} \coloneqq p_{ij}^{(1)} + p_{ij}^{(2)} + \dots + p_{ij}^{(L_{ij})}$. However, the expected number of claims is not necessarily equal to the expected *observed* number of claims, known at the given point of time.[4] In a similar manner to the construction of the expected claims observation frequencies, f_j, in [T11], Subsection 3.3, the a priori expectation should be adjusted to take *non-homogeneous effects over time* and *characteristics specific to the global entity* into account.

Secondly, to withdraw construction-related correlation between the two components of the credibility vector as discussed in Subsections 2.3 and 2.4, the observed *average* claims amount, $\bar{S}_{ij}$, should be compared to the a priori expected *average* claims amount within the individual entity, $\bar{\Pi}_{ij}$, defined by[5]

$$\bar{S}_{ij} = \frac{S_{ij}}{N_{ij}} \qquad \text{and} \qquad \bar{\Pi}_{ij} = \frac{\Pi_{ij}}{p_{ij}} .$$

For a *homogeneous* group of insured lives, the risk measure based on claim severity is, *in expectation*, independent of the observed or expected *number* of claims. Analogously to claim numbers, the expectation should be adjusted.[6]

The claims observations of the individual entity i for the two components may then be defined as follows:

$$R_{ij}^{\mathrm{n}} = \frac{N_{ij}}{p_{ij}} \left(\frac{\sum_{i=1}^{I} N_{ij}}{\sum_{i=1}^{I} p_{ij}} \right)^{-1} \qquad \text{and} \qquad R_{ij}^{\mathrm{s}} = \begin{cases} \dfrac{\bar{S}_{ij}}{\bar{\Pi}_{ij}} \left(\dfrac{\sum_{i=1}^{I} S_{ij}}{\sum_{i=1}^{I} N_{ij}} \right)^{-1} \left(\dfrac{\sum_{i=1}^{I} \Pi_{ij}}{\sum_{i=1}^{I} p_{ij}} \right) & \text{, if } N_{ij} > 0 \\ 1 & \text{, if } N_{ij} = 0 \end{cases} ,$$

where the first component is based on the observed number of claims, and the second component on the observed claims amounts of the individual entity i in relation to the global entity during the year j. These quantities are no longer directly correlated in an obvious manner, but only as described in Subsection 2.1. **In fact, if one would want to take both the observed claims frequency and severity into account, the components could now be multiplied.**

Hence, the individual risk experience vector[7], where P represents the global entity, becomes

$$\vec{B}_i = (B_i^{\mathrm{n}} \ , \ B_i^{\mathrm{s}})' = \left(\frac{\sum_{j \in T} N_{ij}}{\sum_{j \in T} f_j \cdot p_{ij}} \ , \ \frac{\sum_{j \in T} \left(q_j \cdot \Pi_{ij} \right)^{-1} S_{ij} p_{ij}}{\sum_{j \in T} N_{ij}} \right)' \quad \text{with} \quad f_j = \frac{\sum_{i \in P} N_{ij}}{\sum_{i \in P} p_{ij}} , \quad q_j = \frac{\sum_{i \in P} S_{ij}}{\sum_{i \in P} N_{ij}} \cdot \frac{\sum_{i \in P} p_{ij}}{\sum_{i \in P} \Pi_{ij}} .$$

In this expression, we set $B_i^{\mathrm{s}} \coloneqq 1$ in the case where no claims were observed at all, i.e. $\sum_{j \in T} N_{ij} = 0$.

[4] Non-homogeneous effects over time, such as IBNR claims, or the influence of economic cycles, imply systematic differences between the generally expected and currently observed claims. These effects, as well as further characteristics, might be specific to the considered risk group. See [T11], Subsection 3.1, for further reference.

[5] It is suggested that the non-differentiated risk premium of an insured life represents the corresponding expected claim amount. The influence of the risk margin contained in the premium on the risk measure is considered negligible, as it is contained in both the nominator and the denominator of the measure.

[6] In fact, a given group of insured lives is not homogeneous in real-life. For instance, there are few insured lives with a high risk premium, and many with a low one. Therefore, the average risk premium might usually over-estimate the observed average claims amount. In addition, whether a given claim is already known by the insurer might be correlated to the corresponding claim amount. Due to the construction of the final risk measure as a standardised frequency, these effects influence both the numerator and denominator of the measure, and thus can be regarded as being largely offset.

[7] The interested reader is referred to [BG05], p. 180ff, for reference. The used weights are $w_{ij}^{\mathrm{n}} = f_j \cdot p_{ij}$, $w_{ij}^{\mathrm{s}} = N_{ij}$, and are assumed to be known.

Again, we need a weight to combine the individual risk experience with the global risk experience. In a multidimensional credibility model, this is the *credibility matrix* A_i, given by

$$A_i = \tilde{T} \times \left(\tilde{T} + \tilde{S}_i\right)^{-1},$$

where $\tilde{S}_i$ and $\tilde{T}$ are *structural parameter matrices* (addressed further below). On the basis of these quantities, the credibility vector is defined as follows:[8]

$$\vec{\varphi}_i = (\varphi_i^{\mathrm{n}}, \varphi_i^{\mathrm{s}})' = A_i \times \vec{B}_i + (\mathbb{I} - A_i) \times \vec{\mathbb{I}}$$

Once applied to compute $\vec{\varrho}_{g(i)}$ for the risk groups, and second to determine $\vec{\gamma}_i$ for the contracts, the final risk adjustment would be equal to

$$\xi_i = \left(\varrho_{g(i)}^{\mathrm{n}} \cdot \gamma_i^{\mathrm{n}}\right) \cdot \left(\varrho_{g(i)}^{\mathrm{s}} \cdot \gamma_i^{\mathrm{s}}\right).$$

In order to be suitable for new business contracts, an insurer might just rely on components based on observed claims numbers, as the non-differentiated risk premium and claims amounts (i.e. the accumulated value of benefits paid plus the present value of expected future benefits) might be insurer specific.

To achieve this result, we need estimators for the structural parameter matrices, $\tilde{S}_i$ and $\tilde{T}$. Similar to the one-dimensional case (see Subsection 1.2), the following (most generic) are suggested:[7]

$$\tilde{S}_i = \begin{pmatrix} \dfrac{\sigma_{\mathrm{n}}^2}{\sum_{j \in T} f_j \cdot p_{ij}} & 0 \\ 0 & \dfrac{\sigma_{\mathrm{s}}^2}{\sum_{j \in T} N_{ij}} \end{pmatrix} \quad \text{with} \quad \begin{aligned} \sigma_{\mathrm{n}}^2 &= \frac{1}{I} \sum_{i=1}^{I} \frac{1}{n-1} \sum_{j=1}^{n} f_j \cdot p_{ij} \left(R_{ij}^{\mathrm{n}} - B_i^{\mathrm{n}}\right)^2, \\ \sigma_{\mathrm{s}}^2 &= \frac{1}{I} \sum_{i=1}^{I} \frac{1}{n-1} \sum_{j=1}^{n} N_{ij} \left(R_{ij}^{\mathrm{s}} - B_i^{\mathrm{s}}\right)^2. \end{aligned}$$

As the estimators of the 2×2 matrix components of $\tilde{T}$ are quite complex, but correspond to the standard case of the multidimensional Bühlmann and Straub model, we refer the interested reader to, for instance, pages 185-186 in [BG05].

[8] $\mathbb{I}$ is the identity matrix of appropriate dimension, consisting of ones in the leading diagonal and zeroes elsewhere. $\vec{\mathbb{I}}$ is a vector of appropriate dimension, only consisting of unity.

3. Continuous evolution of risk judgments

In this section, the problem of unwanted jumps in the risk judgment due to the limitation of observations to a fixed time period, and the passage of time, is addressed.

3.1 Motivation

Risk differentiation systems that provide risk-adjusted insurance premiums are most generally based on some form of experience rating. It is convenient, and reasonable, to restrict the involved risk experience to an observation period, T, of fixed maximum length. Too long a time period would involve outdated risk information, which might be inappropriate to judge the current state of the group of insured lives. This is especially problematic if one of the popular standard models is used, where the same risk characteristics within the whole observation period is assumed. Further, just a few of all in force contracts might provide the full observation data due to the change between insurers. Furthermore, the acquisition of reliable external underwriting data for new business contracts would even be harder.

As time evolves, a new observation year is added to the observation period T. Simultaneously, if the risk experience of a contract incorporates the maximum number of observation years, then the oldest year drops out of T. This shifting is a natural consequence of all fixed maximum length observation periods. Not surprisingly, and confirmed by actual insurance practice, a certain observation year within the observation period of a given contract might be extraordinary in comparison to the other years. That is, for example, there is an accumulation of observed claims, an extreme claims amount, or a lack of any claims. Eventually, such an extraordinary observation year will reach the end of T, and the subsequent year, it will no longer be taken into account at all. This might cause a huge change in the risk adjustment, and thus in the risk premium, which in most cases is inappropriate.

A widespread approach to avoid such effects is the application of a dampening technique from one year to another. But such a system calls for actuarial supervision on a contractual basis, as one might think of examples where a considerable change in the risk premium is justified (e.g. inaccurate underwriting data). In this section, our aim is to provide an alternative route: risk characteristics usually don't change in an abrupt manner, but it is generally understood that they *do change* over time. As an insurer usually determines the risk premium for a future period, it should not rely too much on risk experience from observation years far in the past, as it might not reflect the current risk characteristics. Nevertheless, such information is not entirely useless, and at the same time, the newest risk information might be the least reliable due to IBNR[9] claims. Therefore, a possible approach could be to model this time dependence, where the influence of a given observation year on another decreases with the distance in time.

In the preceding section, the reader was introduced to *multidimensional credibility theory* in order to model correlation between the components of an observation and estimation vector. Previously, the authors of [F05] and [M11] have introduced an approach to model covariance dependent on *distance,* be it geographically, or in time. In fact, the underlying idea can be applied in such a way, that it is useful in terms of the issue discussed here. The vector components will then represent the discrete time axis, while the observation period from the *model perspective* spans just one single 'year'. As a consequence, the standard estimators for the structural parameters cannot be used, which is also addressed in the next subsection.

[9] IBNR is an acronym for ‚incurred but not yet reported'.

3.2 Multidimensional credibility to model time dependence

Rather than combining the results from Section 2 with the subsequently suggested methodology, we here aim to focus on the modelling of time dependence on a stand-alone basis. Therefore, the prerequisite to the following is again given by [T11], of which a short summary can be found in Subsection 1.2. We will make use of the claims observations, C_{ij}, the volume measure, V_{ij}, and the expected claims observation frequencies, f_j, of the individual entity i during the observation year $j \in T$.

As stated in the preceding subsection, multidimensional credibility theory is used to consider the individual risk experience of different years within the observation period as the components of a vector. To remain within a similar approach to what was presented in [T11], we rely on the multidimensional credibility model of Bühlmann and Straub. Similar to Subsections 2.2 and 2.5, the individual risk experience vector is therefore given by

$$\vec{B}_i = \left(B_i^1, \dots, B_i^j, \dots, B_i^n\right)' \in \mathbb{R}^n ,$$

where $n = |T|$ is the number of years within the observation period, with n being the most current year. As in the one-dimensional case, the components are defined as

$$B_i^j = \frac{C_{ij}}{f_j \cdot V_{ij}} ,$$

resulting in the credibility vector

$$\vec{\varphi}_i = \left(\varphi_i^1, \dots, \varphi_i^j, \dots, \varphi_i^n\right)' = A_i \times \vec{B}_i + (\mathbb{I} - A_i) \times \vec{\mathbb{I}} \qquad \text{with} \qquad A_i = \tilde{T} \times \left(\tilde{T} + \tilde{S}_i\right)^{-1} .$$

$\tilde{S}_i$ and $\tilde{T}$ are the structural parameter matrices, given by

$$\tilde{S}_i = \begin{pmatrix} \frac{\sigma_1^2}{f_1 \cdot V_{i1}} & & & & 0 \\ & \ddots & & & \\ & & \frac{\sigma_j^2}{f_j \cdot V_{ij}} & & \\ & & & \ddots & \\ 0 & & & & \frac{\sigma_n^2}{f_n \cdot V_{in}} \end{pmatrix} \quad \text{and} \quad \tilde{T} = \begin{pmatrix} \tau_{11} & \cdots & \tau_{1j} & \cdots & \tau_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tau_{j1} & \cdots & \tau_{jj} & \cdots & \tau_{jn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tau_{n1} & \cdots & \tau_{nj} & \cdots & \tau_{nn} \end{pmatrix} .$$

Similar to the simpler case outlined in Subsection 1.2, the model can be applied in terms of a two-layer approach. As a suggestion, the global layer does not need to be considered multidimensional, resulting in the risk level $\varrho_{g(i)}$ of the risk group in relation to the insurer's whole portfolio.

However, the final target is again to determine the risk adjustment ξ_i of contract i, which should be in line with the *current* risk characteristics of the group of insured lives. The last component, $\gamma_i \coloneqq \varphi_i^n$, of the credibility vector represents an estimator based on the latest risk observation, which in turn incorporates all the former risk observations via correlation. It is then reasonable to define

$$\xi_i \coloneqq \varrho_{g(i)} \cdot \gamma_i \quad .$$

The treatment of new business, as well as some further specialties, have already been discussed in [T11].

3.3 Constitution of the structural parameter matrices

It was argued before, that the standard estimators for the structural parameter matrices of Bühlmann and Straub cannot be used. The reason is that, similar to the one-dimensional case, the estimation of the diagonal elements of $\tilde{S}_i$, notably σ_j^2, $j \in \{1, \dots, n\}$, involves several observations (e.g., the years within an 'observation period') for each of the components. From a model perspective under this modified approach, we do not have several observations of the same risk for a given component, such as B_{ik}^j, $k \in \{1, \dots, m\}$. Rather, the time axis is now arranged as components of the multidimensional vector, and there is no deviation in a single observation.

However, in our application we 'know' about the uncertainty of such an observation from the one-dimensional model, where we consider the observations over time. In particular, the multidimensional approach, given the corresponding circumstances, should lead to the same results as the one-dimensional model. Hence, it is worth considering 'full correlation' in the multidimensional approach, where all the components of the individual risk experience vector $\vec{B}_i$ (e.g., the different observations in time) are fully correlated. This is, for instance, given by the special case where

$$\tilde{S}_i = \sigma^2 \cdot \begin{pmatrix} (f_1 \cdot V_{i1})^{-1} & & & & 0 \\ & \ddots & & & \\ & & (f_j \cdot V_{ij})^{-1} & & \\ & & & \ddots & \\ 0 & & & & (f_n \cdot V_{in})^{-1} \end{pmatrix} \quad \text{and} \quad \tilde{T} = \tau^2 \cdot \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{pmatrix},$$

resulting in the credibility vector

$$\vec{\varphi}_i = \varphi_i \cdot \vec{\mathbb{I}} \; .$$

In other words, 'full correlation' between the years of the observation period results in the known one-dimensional credibility estimator of Subsection 1.2 for all vector components. Indeed, in line with the mindset of the one-dimensional credibility model of Bühlmann and Straub, where the risk characteristics of the individual entity under consideration remains the same within the observation period, one might expect this result. Therefore, one might suggest defining the structural parameters of the main diagonal within the parameter matrices of Subsection 3.2 to be the ones from the one-dimensional case, that is firstly

$$\sigma_j^2 := \sigma^2 \qquad \text{and} \qquad \tau_{jj} := \tau^2 \qquad \forall j \in \{1, \dots, n\} \; .$$

Secondly, in its most general form,

$$\sigma^2 = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{n-1} \sum_{j=1}^{n} w_{ij} \left(B_i^j - B_i \right)^2,$$

where

$$B_i = \sum_{j=1}^{n} \frac{w_{ij}}{w_{i*}} B_i^j \,, \qquad w_{ij} = f_j \cdot V_{ij} \qquad \text{and} \qquad w_{i*} = \sum_{j=1}^{n} w_{ij} \; .$$

And thirdly,

$$\tau_{jj} = \tau^2 = \max\left[0\,,\, c\cdot\left\{\frac{I}{I-1}\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}(B_i-\bar{B})^2 - \frac{I\cdot\sigma^2}{w_{**}}\right\}\right] \quad \forall j\in\{1,\dots,n\}\ ,$$

where again

$$c = \frac{I-1}{I}\left\{\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}\left(1-\frac{w_{i*}}{w_{**}}\right)\right\}^{-1}, \qquad \bar{B} = \sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}B_i\ , \qquad \text{and} \qquad w_{**} = \sum_{i=1}^{I}\sum_{j=1}^{n} w_{ij}\ .$$

The remaining matrix elements of $\tilde{T}$, i.e. τ_{jk} with $j \neq k$, are key to the modelling of time dependence but were not yet discussed. As described in Subsection 3.1, the idea is to allow for correlation between the observations of the years within the observation period T, where τ_{jk} represents the covariance of the risk characteristics of the years j and k.

The closer two years are located one after another, i.e. the smaller $|j-k|$, the larger their correlation. So distance in time between two observations should be taken into account. Additionally, it is unlikely that zero correlation is ever reached between two observations, even after a long time.

Following the construction of the risk measures, where we took non-homogeneous effects over time into account, it seems a reasonable assumption that any correlation between two observations is stationary with respect to the time, i.e.

$$\tau_{j,j+d} = \tau_{k,k+d} \qquad \forall j,k\in\{1,\dots,n\} \quad \forall d\in\{1,\dots,n-\max[j,k]\}\ .$$

These properties motivate an exponential coherence with respect to the distance in time. Thus, as illustrated in Figure 5, it is suggested that

$$\tau_{jk} = \delta^{|j-k|}\cdot\tau^2 \qquad \forall j,k\in\{1,\dots,n\} \quad \text{with} \quad 0<\delta<1\,.$$

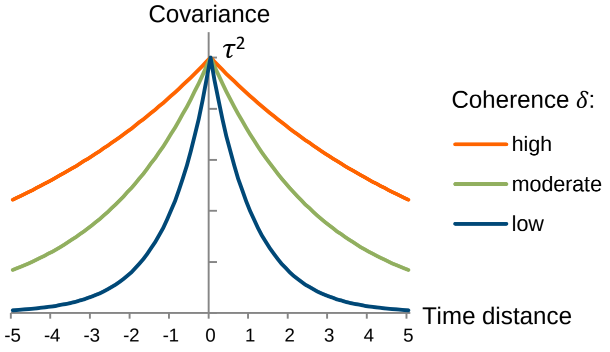


Figure 5: Influence of neighboring observation years

The coherence parameter δ could, for example, be determined according to some *pure Bayesian approach* (to incorporate an ‘expert’s opinion’). From a statistical perspective, an insurer might also estimate δ from its portfolio by using standard correlation estimation techniques on the set (B_i^j, B_i^{j+1}), $j\in\{1,\dots,n-1\}$, $i\in\{1,\dots,I\}$.

4. Poor underwriting quality

This section is about individual contracts coming onto the books of a given insurer with poor underwriting data, such that the application of the usual risk differentiation system might lead to systematically wrong judgments. In the following section, a possible model modification and its applications are discussed.

4.1 Motivation

The risk premium an insurer should demand from a group of insured lives to cover the insured risks is certainly affected by many different parameters. There are sophisticated pricing regimes to include a large number of measurable tariff criteria, and to provide a realistic representation of business reality. Consequently, with an increasing number of analysis factors under consideration, the need for sufficient and reliable underwriting data will also grow, for both business in force and new business.

Today, risk differentiation models usually presume the availability of such underwriting data and its sound quality, be it at least within the business in force. Depending on the individual contract, the insurer's situation, the given industry, or the insurance market concerned, this assumption might be far from reality. One may think of the following examples, where the quality of underwriting data is questionable:

- **New business contracts.** The risk judgment of a new business contract is usually based on external underwriting data. There might be incentives for the previous insurer or the insured company to artificially 'beautify' the risk situation, or to supply only limited risk data in order to avoid cost and time effort. In an extreme case, there is certainly an important difference between the (reliable) information about no claims experience, and the lack of information about claims experience.

- **Introduction of a new pricing regime.** An insurer might want to introduce a more sophisticated or adequate pricing regime for a certain insurance product. Previously, it might not have been necessary to store detailed risk information data on a contractual basis for a sufficiently large time period. Consequently, the insurer might, for instance, not even be able to distinguish between mortality and disability claims of the past. This affects the statistical significance of the individual risk experience for a subset of the insurer's portfolio, or range of products palette.

- **Industry traditions.** Most present in emerging insurance markets, some industry sectors have a long tradition of insurance demand, which might even be anchored in legal regulations. If the need for group life insurance increases within other industry sectors, or if it even becomes mandatory, an insurer might experience a considerable heterogeneity of the underwriting quality in its portfolio.

- **Mergers and acquisitions.** Similar to the aforementioned industry traditions, mergers and acquisitions activities might result in sub-portfolios with different availability and quality of risk data.

- **Microinsurance markets.** Providing insurance products in growing microinsurance markets is challenged by fundamental issues. Constraints on data availability and reliability, and the low willingness to cooperate with insurers, leads to a significant pricing risk. This situation might even be dependent on the volume of a given contract, i.e. there might be reliable underwriting data available for a large company, whilst there is poor information about a small company.

Poor underwriting quality might result in several unwanted consequences, especially if used in a risk differentiation model that does not take systematic uncertainties into account. Partial non-competitiveness, inadequate cross-financing, and adverse selection are some examples. And if withholding of underwriting data for a new business contract generally leads to a lower risk premium in comparison to a customer who is willing to deliver claims data transparently, the situation won't get any better.

Certainly, a minimum level of data availability and reliability for at least a large proportion of an insurer's portfolio is necessary to provide risk-adjusted pricing. In the following subsection, we outline a possibility to handle poor underwriting quality in a minor subset of an insurer's portfolio, given the context of [T11]. If, however, the whole portfolio is affected, one may find an appropriate approach detailed in Section 5.

4.2 Adjustment of the credibility weight

Under the aforementioned assumed circumstances, we assume that the global risk experience (be it either the risk group, or the whole portfolio) is reliable. But the poor underwriting quality affects the statistical significance of the individual risk experience, which is used to estimate the risk level of the given contract in relation to the corresponding risk group. Therefore, within a two-layer approach, the model construction within the lower layer is to be investigated.

First of all, the manner in which statistical significance – or, conversely, uncertainty – of the individual risk experience is incorporated in the Bühlmann and Straub credibility model should be considered. To do so, one has to think of the individual entity i being characterised by its risk profile ϑ_i, which itself is the realization of a random variable Θ_i. Once given ϑ_i, the individual risk observations R_{ij} are randomly and independently drawn according to a distribution function F_{ϑ_i}.[10] The resulting credibility estimator of the model is then an approximation of $E\left[R_{ij}|\Theta_i\right]$.

Within this approach, the standard conditional variance assumption is

$$Var\left(R_{ij}|\Theta_i\right) = \frac{\sigma^2(\Theta_i)}{w_{ij}}$$

where $\sigma^2(\Theta_i)$ represents the uncertainty within the individual risk, with $\sigma^2 = E[\sigma^2(\Theta_i)]$, and w_{ij} is the known volume measure of the individual entity i in the year j.

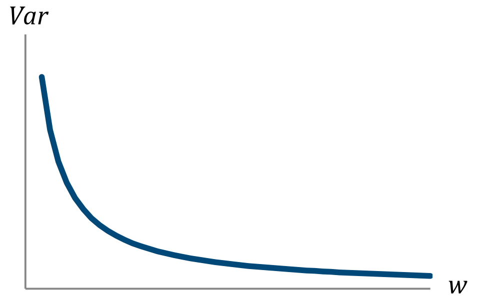


Figure 6: Variance in standard assumption

With reference to the standard estimator for σ^2, as pointed out in Subsection 1.2, the aforementioned variance is often understood to be the volatility of an individual risk experience in time. However, more accurately, it is a measure for the uncertainty within the compressed risk experience, R_i, of the single observations R_{ij}. As such, it also directly impacts the credibility weight, α_i, which is given to the individual risk experience in relation to the global risk experience. Poor underwriting quality, as described in the preceding Subsection 4.1, affects the reliability of the observed risk experience, and will thus affect the credibility weight, α_i, too.

Therefore, it is suggested to take account of poor under-writing data with a further variance component in addition to the standard assumption, which is specific to the given individual entity:

$$Var\left(R_{ij}|\Theta_i\right) = \frac{\sigma^2(\Theta_i)}{w_{ij}} + v_i$$

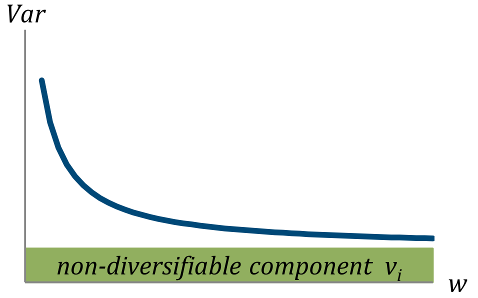


Figure 7: Variance in the modified assumption

[10] This concept is often referred to as *two-urn model* (see, for instance, [BG05] for a more detailed introduction).

If $\nu_i \geq 0$ is a constant, then it represents the non-diversifiable part of the uncertainty within an individual risk observation, as shown in Figure 7. But there are situations where it is reasonable to assume ν_i is volume dependent. This is discussed later.

Consequently, the credibility estimator has to be re-evaluated. Its shape and properties remain the same, but the credibility weight – to be computed by a minimisation argument on the expected squared error with respect to the Bayes estimator[11] – changes as follows:

$$\tilde{\alpha}_i = \frac{w_{i*}}{w_{i*} + \dfrac{\sigma^2 + \delta_i}{\tau^2}} \quad , \quad \text{where} \quad \delta_i = \frac{\nu_i}{w_{i*}} \sum_{j=1}^{n} w_{ij}^2 \geq 0 \quad \text{and} \quad w_{i*} = \sum_{j=1}^{n} w_{ij} \ .$$

If $\nu_i = 0$, the known credibility weight α_i from the Bühlmann and Straub model results. Clearly, for $\nu_i > 0$ the modified credibility weight is smaller, i.e. $\tilde{\alpha}_i < \alpha_i$, reflecting the fact that the statistical significance of the individual risk experience is lower. In terms of the context in Subsection 1.2, and [T11], the weight w_{ij} is given by the 'observation related volume', $w_{ij} = f_j \cdot V_{ij}$.

According to the prerequisite from Subsection 4.1, only a minor part of the insurer's whole portfolio should be significantly influenced by poor underwriting quality (otherwise, the reader is referred to Section 5). Hence, the structural parameters, f_j, σ^2 and τ^2, of the model can still be estimated from the insurer's portfolio, potentially by excluding the relevant sub-portfolio when applying the statistics. Consequently, there is no modification to the estimators required for these quantities.

Finally, it has not yet been discussed how to determine the additional variance component ν_i. Indeed, it is a challenging task to quantify a parameter based on information that is *explicitly unknown*. One might think of an approach where for new business, for instance, the previous external underwriting data is compared to the current risk information, which has been observed by the insurer. But this would only hold true in a world where risk characteristics of a group of insured lives do not change. Further, it is doubtful whether such a quantity could be applied to other contracts, since, for the contract under consideration, it is too late. Following a different approach, one could compare several affected new business contracts with a similar sub-portfolio of the business in force to quantify ν_i, e.g. of the same corporate group. After all, these thoughts just cover a small portion of the possible scenarios from Subsection 4.1.

If no statistical methods are convenient to determine the parameter, it is suggested to follow a so-called *pure Bayesian approach* (to incorporate an 'expert's opinion'), which could also be understood as the application of a *management tool* (to incorporate a 'management decision'). The idea is to transform the process of estimating a technical model parameter into the setting of a quantity that is intuitively understood. It might seem questionable to apply this approach for a single affected contract (except from a didactical reason). But given that an insurer was able to quantify the affliction by poor underwriting quality in a certain contract, or sub-portfolio, it might be reasonable to allow for the 'same' impairment on further contracts or sub-portfolios. This would preserve the properties of the risk-differentiation system.

In the above formula, the relation between ν_i and $\tilde{\alpha}_i$ is shown. There are several ways now to exploit this link to determine ν_i. In the following subsections, some general suggestions are provided to treat the scenarios that were outlined in Subsection 4.1.

[11] The calculations supporting this result can be found in the Appendix.

4.2.1 Direct definition of the credibility weight

Given a specific contract i (be it new business, or business in force), one might *directly decide the weight* $\tilde{\alpha}_i$ that should be given to its individual risk experience, R_i, in contrast to the weight $(1-\tilde{\alpha}_i)$ of the global experience. This would require that, for some reason, the specific impact of poor underwriting quality to this contract is quantifiable. The resulting risk judgment is then, as described in Subsection 1.2,

$$\varphi_i = \tilde{\alpha}_i \cdot R_i + (1-\tilde{\alpha}_i) \cdot 1 \ .$$

The insurer now comes across another contract k, possibly with different volume and in a different risk group, and one is convinced that it is afflicted by the same kind of poor underwriting quality (for example, due to the same former insurer, the same management system, or the same regulatory or local restrictions). Hence the above information can be used by rewriting the formula for $\tilde{\alpha}_i$:

$$\nu_k \coloneqq \nu_i = \frac{w_{i*}}{\sum_{j=1}^{n} w_{ij}^2}\left\{\tau^2 w_{i*}\left(\frac{1}{\tilde{\alpha}_i}-1\right)-\sigma^2\right\}$$

By again using the formula for $\tilde{\alpha}_k$ with all the parameters of the second contract k, the desired result φ_k can be computed, which takes the same (relative) uncertainty due to poor underwriting quality into account.

4.2.2 Specification of the credibility coefficient

Similar to the preceding description, one might be able not to directly determine the credibility weight for a given contract, but to *quantify a certain volume* at which the weighting between the individual and the global risk experience should be equal for relevant contracts, i.e. $\tilde{\alpha}_i = 50\%$. With respect to the formula for $\tilde{\alpha}_i$, this is given if the quantity w_{i*} is equal to the credibility coefficient, $\tilde{\kappa}$, defined by

$$\tilde{\kappa} = \frac{\sigma^2 + \delta_i}{\tau^2}.$$

As in Subsection 1.2, and [T11], it holds true that $w_{ij} = f_j \cdot V_{ij}$. Given the desired (constant) volume, V, of a fictitious contract with the above property, this results in

$$\nu_i = \frac{\left(\sum_{j=1}^{n} f_j\right)^2}{\sum_{j=1}^{n} f_j^2}\left\{\tau^2 - \frac{\sigma^2}{V \cdot \sum_{j=1}^{n} f_j}\right\} \ .$$

Obviously, this procedure requires V to be sufficiently large, such that ν_i becomes non-negative.

4.2.3 Comparison between the standard and the modified model

The possibly most applicable approach in practice is to compare both the modified and the original model, where poor underwriting quality does not occur. One might then agree on the fact that a certain type of assumed reservations about underwriting quality, given a particular volume of a contract, would lower the credibility of the contract's individual risk experience by a certain percentage. For example, if an external portfolio with doubtful data management is merged into an insurer's own portfolio, the insurer might decide to trust the individual risk experience of contracts with 100 insured lives at 20% less than usual.

To formalize these thoughts, let $\tilde{\alpha}_i$ be the modified and α_i the non-modified credibility weight. Due to the influence of poor underwriting quality, these quantities are linked as follows:

$$\tilde{\alpha}_i = q \cdot \alpha_i \,, \quad q \in (0,1] \,,$$

where in the above example we had $q = 0.80$ at a (constant) volume of $V_{ij} = 100$, $j = 1, \dots, n$, insured lives. After some calculation, the additional variance component results as

$$\nu_i = \frac{w_{i*}}{\sum_{j=1}^{n} w_{ij}^2} \left(\frac{1}{q} - 1 \right) (\tau^2 w_{i*} + \sigma^2) \,, \qquad \text{where} \qquad w_{ij} = f_j \cdot V_{ij} \,.$$

4.2.4 Volume dependence

As formerly mentioned, there are situations where it is reasonable to assume ν_i are volume dependent. A typical example might be microinsurance markets. In such a circumstance, large companies might more likely be insured by a large insurer, and also be more able to perform proper underwriting and data management. Consequently, underwriting quality might be seen to be better for large volume contracts, and worse for small entities.

One could simply formalise this link by, for example, setting $\nu_i \coloneqq \mu_i / w_{ij}$, where again $w_{ij} = f_j \cdot V_{ij}$. This approach can be used with any of the preceding ideas outlined in Subsections 4.2.1, 4.2.2, and 4.2.3.

5. Permanent change in risk characteristics

This section proposes a preliminary stage to an evolutionary credibility model, where a permanent change in risk characteristics within an observation period is taken into account.

5.1 Motivation

The use of an observation period, rather than just a single year's observation, bears many advantages in terms of, for instance, statistical stability and comparability. Nevertheless, a large fraction of the most popular risk differentiation systems – not only in the domain of credibility theory – assumes that an individual entity's risk characteristics remained the same during the entire observation period. So does the renowned credibility model of Bühlmann and Straub, as well as the modified model approach presented in [T11] and Subsection 1.2.

As one might think, this assumption is somewhat simplistic in many real-world applications. Though random deviations in risk observations are handled by the structural parameters addressed in the aforementioned Subsection 1.2, systematic changes due to mutations in a company's personnel (e.g. hires, resignations, and retirements) or other influences are not taken into account. Therefore, the statistical significance of individual risk experiences assumed by the model might be wrong, and neither the estimated risk structure within risk groups, nor the whole portfolio might reflect the reality.

One might notice the similarity to Section 4 concerning poor underwriting quality, where also the credibility of the individual risk experience is concerned. Indeed, the two issues can be considered by the same modification of the model, as further explained in Subsection 5.2. However, as the insurer's whole portfolio is affected by a permanent change in risk characteristics rather than just a single contract or a small sub-portfolio, there are further consequences to the model and its parameters.

5.2 Amendment of the parameter estimators

As per Subsection 4.2, the reader should familiarise oneself with the incorporation of uncertainty within the individual risk experience in the Bühlmann and Straub credibility model. In the following, we will make use of the same notation, i.e. the risk profile ϑ_i as realization of Θ_i, the individual risk observations R_{ij} of individual entity i in the year j as draws according to the distribution F_{ϑ_i}.[10]

Again, the standard conditional variance assumption no longer holds true, as, due to systematic volatility within risk observations from one year to another, there is a non-diversifiable variance component. This is illustrated in Figure 7 in Subsection 4.2. In the most general approach, one might now argue that the same modification of the model – with an *individually determined* variance component ν_i – should be considered, as there are groups of insured lives with more systematic changes in risk characteristics than others.

However, if one considers an insurer's whole portfolio as affected by such changes, and the *individual impact* on each of the contracts is not entirely known, then one might fail no later than at estimating the structural parameters of the credibility model. Moreover, there is little practical benefit in using too many model parameters (and this idea would introduce as many parameters as we have contracts). Hence, it is reasonable to capture an average systematic volatility within the same risk group (or, respectively, global entity), and one could think of this variance component as 'volatility margin' in some sense. The model assumption about conditional variance in individual observations will then become

$$Var\left(R_{ij}\middle|\Theta_i\right) = \frac{\sigma^2(\Theta_i)}{w_{ij}} + \nu \ ,$$

where $\sigma^2(\Theta_i)$ represents the uncertainty within the individual risk, with $\sigma^2 = E[\sigma^2(\Theta_i)]$, and w_{ij} is the known volume measure of the individual entity i in the year j.

Consequently, the formula for the credibility weight changes the same way as in Subsection 4.2:

$$\tilde{\alpha}_i = \frac{w_{i*}}{w_{i*} + \frac{\sigma^2 + \delta}{\tau^2}} \;, \quad \text{where} \quad \delta = \frac{\nu}{w_{i*}} \sum_{j=1}^{n} w_{ij}^2 \geq 0 \quad \text{and} \quad w_{i*} = \sum_{j=1}^{n} w_{ij} \;.$$

At first sight, there appears no difference to the former result of Subsection 4.2. But now that the whole portfolio is affected by an additional variance component, the standard estimators for the structural parameters σ^2 and τ^2 cannot be applied any more. In addition, an estimation procedure for ν is needed.

5.2.1 Estimation of σ^2 and ν

In the Bühlmann and Straub model[12] the variance within the individual entity i is considered by

$$S_i = \frac{1}{n-1} \sum_{j=1}^{n} w_{ij} \cdot \left(R_{ij} - R_i\right)^2 \;,$$

where all quantities are defined as per Subsection 1.2. Without model modification, this expression has expectation $E[S_i] = \sigma^2$, and thus the average within the whole global entity is used as estimator for σ^2. However, due to the model modifications, in the present context the expectation changes to the following:[13]

$$E[S_i] = \sigma^2 + c \cdot \overline{w}_{i*} \cdot \nu \;,$$

where

$$\overline{w}_{i*} = \frac{1}{n} \sum_{j=1}^{n} w_{ij} \;, \qquad c = \frac{n}{n-1} \sum_{j=1}^{n} \frac{w_{ij}}{w_{i*}} \left(1 - \frac{w_{ij}}{w_{i*}}\right) \qquad \text{with} \qquad \begin{cases} c = 1, & \text{if } w_{ij} = \text{const} \quad \forall j \\ c < 1, & \text{else} \end{cases} \;.$$

In expectation, the observation S_i of each individual entity i fulfils this equation, where S_i and $c \cdot \overline{w}_{i*}$ are known, but σ^2 and ν to be estimated. Consider these pairs $(c \cdot \overline{w}_{i*}\,, S_i)$ as points in a Cartesian coordinate system. Now, the above equation can be interpreted as a line with slope ν and axis intercept σ^2. Thus we compute a regression line through the points $(c \cdot \overline{w}_{i*}\,, S_i)$, $i = 1, \dots, I$, to estimate these parameters, as illustrated in the following figure:

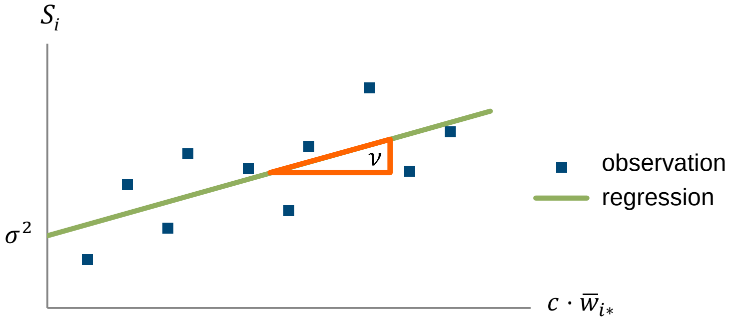


Figure 8: Regression line for parameter estimation

[12] See [BG05], Subsection 4.8, for reference.

[13] The calculations supporting this result can be found in the Appendix.

In the context of [T11] and Subsection 1.2 we have again $w_{ij} = f_j \cdot V_{ij}$.

5.2.2 Estimation of τ^2

Similar to the preceding approach, we consider the variance within the global entity by

$$T = \frac{I}{I-1} \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} (R_i - \bar{R})^2$$

with notation according to Subsection 1.2. After some computation, we obtain[14]

$$E[T] = \frac{I \cdot \sigma^2}{w_{**}} + \frac{I \cdot \nu}{I-1} \sum_{i=1}^{I} \sum_{j=1}^{n} \frac{w_{ij}}{w_{**}} \left(\frac{w_{ij}}{w_{i*}} - \frac{w_{ij}}{w_{**}} \right) + \frac{I \cdot \tau^2}{I-1} \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} \left(1 - \frac{w_{i*}}{w_{**}} \right) ,$$

where again, *in expectation*, the observation T fulfils the equation. Thus, by using the estimators for σ^2 and ν from Subsection 5.2.1, the following estimator for τ^2 results:

$$\tau^2 = \max \left[0 \, , c \cdot \left\{ \frac{I}{I-1} \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} (R_i - \bar{R})^2 - \frac{I \cdot \sigma^2}{w_{**}} - d \cdot \nu \right\} \right] ,$$

where

$$c = \frac{I-1}{I} \left\{ \sum_{i=1}^{I} \frac{w_{i*}}{w_{**}} \left(1 - \frac{w_{i*}}{w_{**}} \right) \right\}^{-1} \geq 0 \quad \text{and} \quad d = \frac{I}{I-1} \sum_{i=1}^{I} \sum_{j=1}^{n} \frac{w_{ij}}{w_{**}} \left(\frac{w_{ij}}{w_{i*}} - \frac{w_{ij}}{w_{**}} \right) \geq 0 ,$$

with

$$\begin{cases} c = 1, & \text{if } w_{i*} = \text{const } \forall i \\ c < 1, & \text{else} \end{cases} \quad \text{and} \quad d = \frac{1}{n}, \ \text{if } w_{ij} = \text{const } \forall i \, \forall j \ .$$

[14] The calculations supporting this result can be found in the Appendix.

6. Further issues

This section considers four further issues motivated by actual insurance business, grouped into correspon-ding subsections. The first is about including external risk information in the risk-judgment, the second considers limitations regarding the incorporation of individual risk information in the pricing, and the third discusses the more separated risk judgment of large volume contracts in an insurer's portfolio. Finally, the handling of outlier observations within the observation period is addressed in the last subsection.

6.1 Inclusion of external risk information

In the preliminary research report, [T11], the reader was introduced to a risk differentiation model where risk adjustments are computed from past risk observations of a given contract. Moreover, the methodology is based on a certain model structure, with parameters to be estimated from the insurer's own portfolio, and underwriting data to be captured according to the given pattern (even if externally provided in the case of new business).

The situation might arise where an insurer is aware of additional, external information about the *true current* (or *near future*) risk level of a certain contract, which might not even fit the given input pattern of claims observations within the insurer's pricing model. Typically, there has been a fundamental change in the circumstances of an insured group of lives, which has not been settled within the risk observations of the past, but gives rise to the need for an adjustment of the future risk premium. Examples are:

- The company **redefined its business objectives** but is still allotted to the same risk group. The insurer therefore assumes the individual risk judgment to tend towards a higher or lower level.
- The group of insured lives within a contract was significantly changed due to the involvement of the insured company in **merger and acquisition** activities. Risk information about the additional or the remaining sub-portfolio is nevertheless available, and the insurer might want to avoid an abrupt change in risk premiums.
- A **new management regime** was introduced, causing a different organisational structure, changes in employment conditions, or personnel.
- The insurer uses **any other information source** about the risk level of a contract, where this data is only reliable to a certain degree.

Usually, an insurer might consider applying adjustments to the risk premium of an affected contract by manual intervention on the basis of the opinion of an actuarial or other expert. This subsection is, in contrast, about the incorporation of such *exogenous risk information* in a more structured manner. As a prerequisite, it is necessary to quantify both the expected target risk level of the contract and the corresponding uncertainty of this information.

Under the circumstances described in Subsection 1.2, assume there is a random variable Z representing the postulated target risk level of the considered individual entity in relation to the corresponding global entity, where this information is exogenous to the model and is caused by any of the aforementioned examples.[15] Without modification, the credibility estimator of contract i (the individual entity) is

$$\varphi_i = \alpha_i \cdot R_i + (1 - \alpha_i) \cdot R\,, \quad \text{where} \quad R = 1\,.$$

[15] The interested reader is referred to [T08], Sections 3 and 4, for a thorough derivation of the concept.

The global risk experience used in this estimator is $R = 100\%$, meaning the risk level of the global entity in relation to itself, and the individual risk experience is, in average, expected to match this value. Assuming that the contract's risk behaviour will on average tend from the current global risk level to a 10% lower level, one might then use as exogenous information, in relation to the underlying global entity[16], $Z = 0.9$.

The credibility estimator is then amended to

$$\tilde{\varphi}_i = \tilde{\alpha}_i \cdot R_i + (1 - \tilde{\alpha}_i) \cdot Z \,,$$

where the random variable Z replaces the global risk experience, $R = 1$. Prior to presenting the impact on the credibility weight α_i, one has to quantify the uncertainty of Z. From a model perspective, this is formalized by

$$\zeta^2 \coloneqq Var(Z) \,,$$

which, in the same time, is equal to the variance coefficient (as we globally assume $E[Z] = 1$). It might seem artificial to determine ζ^2 without further information, but at least this parameter has a less direct impact on the credibility estimator $\tilde{\varphi}_i$ than modifying it manually upon some belief. It is also possible to quantify ζ^2 in terms of the structural parameter τ^2, by analysing the impact on the credibility weight $\tilde{\alpha}_i$ of the individual risk experience R_i (compare to Subsection 1.2). Due to the inclusion of the exogenous information, the credibility weight changes as follows:

$$\tilde{\alpha}_i = \frac{w_{i*}}{w_{i*} + \dfrac{\sigma^2}{\tau^2 + \zeta^2}} \,, \quad \text{where} \quad w_{i*} = \sum_{j \in T} f_j \cdot V_{ij} \,.$$

Therefore, it is apparent that the incorporation of Z increases the weight of the (unchanged) individual risk experience R_i, giving the residual weight to the exogenous information Z. It should also be intuitively clear, that the more Z deviates from the original global risk experience $R = 1$, the larger its uncertainty ζ^2 should be (and thus, the smaller the corresponding weight). Finally, using $Z = R$ with uncertainty zero leads to the unchanged model as per Subsection 1.2.

Altogether, as time evolves, the cause for the exogenous information might also have settled within the risk observations of the observation period. If so, this approach requires a reduction in the impact of Z as soon as the impact of the initial cause starts to become observable within R_i. The same occurs, if the belief in the actual impact of Z vanishes.

6.2 Limited individual pricing

The fundamental paradigm underlying credibility theory is to combine a *global risk experience* (unspecific, generic) and the *individual risk experience* (specific, distinct) of a certain entity according to their corresponding statistical significance. If the volume of a contract (e.g., the number of insured lives of a group) provides an indicator for the statistical significance of its corresponding claims observations, then risk judgments of large volume contracts would mainly rely on their individual risk experience, whilst for small volume contracts, the global risk experience would dominate.[17]

The weighting between the two sources of information is *linear*, and it is *smooth* with respect to the volume measure of the contract. In fact, in the Bühlmann and Straub model, there is no possible volume at which an entity is solely judged upon either the global (i.e. the corresponding weight is $(1 - \alpha_i) = 1$), or the individual risk experience (i.e. the corresponding weight is $\alpha_i = 1$).

[16] In our example, the exogenous information about the individual entity needs to be quantified in relation to the global entity, as the credibility estimator itself represents the risk level of the individual entity in relation to the global entity.

[17] See Section 1 for an introduction on the given circumstances.

Nevertheless, business reality might again unveil different needs. One may think of examples, where the incorporation of individual risk experience in the risk judgment is desired only where the volume measure exceeds a given limit:

- The assessment of individual risk information about a group of insured lives might be **complex and/or expensive**, though the global risk information is more readily available. At the same time, given a small volume of the contract, individual risk information is allotted a low weight anyway. Hence, an insurer might consider avoiding such an assessment for volumes that lead to a credibility weight α_i smaller than, say, 15%.

- In some countries, the direct incorporation of individual risk experience within the differentiated risk premium is **legally regulated** for group life business. In such cases, the risk premium of contracts with a volume of up to, for example, 1,000 insured lives has to rely on global risk experience only (for example, according to a risk group including certain industry sectors). If an insurer nevertheless would like to use credibility theory for risk-adjusted pricing, there is a large 'jump' in the risk premium to be expected by exceeding the volume limit.

Such an artificial limit does not combine well with the principles of credibility theory. Rather than just applying either a 'simple risk differentiation' for small contracts, or a full 'credibility model-based risk differentiation' for large contracts, an insurer might want to (or have to) dovetail both models to control the extent of the 'jump'. If not, fairness of the risk adjustment of any contract with volume near the limit might be doubtful. In addition, there might be actuarial issues arising in terms of adverse selection or moral hazard risks.

6.2.1 Floor of small credibility weights

In the aforementioned case, where individual risk assessment for *small* contracts is avoided, change in the underlying model might not even be necessary. Not many claims are likely to be observed for a small contract anyway, and so the weighted impact of individual risk experience on the final risk judgment is very limited, given a credibility weight $\alpha_i < L$, where $L \in (0,1)$ is near to zero. Thus it might be reasonable to set α_i to zero for values less than L, as illustrated in Figure 9.

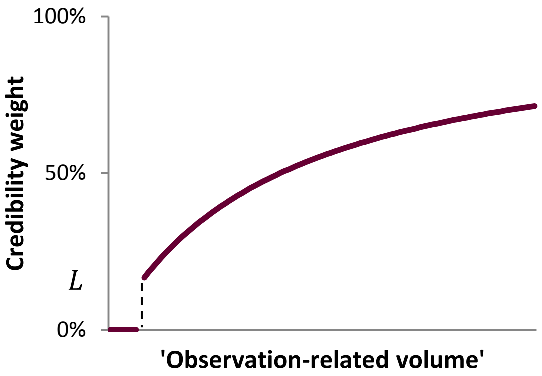


Figure 9: Floor of the credibility weight

In the context of the model of [T11], along with Section 1.2, the credibility weight is defined as:

$$\alpha_i = \frac{\sum_{j \in T} f_j \cdot V_{ij}}{\sum_{j \in T} f_j \cdot V_{ij} + \frac{\sigma^2}{\tau^2}}$$

Therefore, the (constant) volume V_i^L of a contract up to which no individual risk experience is needed, is given by

$$V_i^L = \frac{1}{\sum_{j \in T} f_j} \cdot \frac{L}{1-L} \cdot \frac{\sigma^2}{\tau^2} \,,$$

resulting in the corresponding risk level

$$\varphi_i = \begin{cases} R & , \ \text{if } V_i < V_i^L \\ \alpha_i \cdot R_i + (1-\alpha_i) \cdot R \,, & \text{else} \end{cases} .$$

6.2.2 Adjustment of the credibility coefficient

In the aforementioned case, where the incorporation of individual risk experience starts just from a considerable large contract volume, the above procedure seems unreasonable. Depending on the volume limit, the impact of the individual risk information on the credibility estimator might be significant. Further, it is questionable whether the estimation of the structural parameters of the model based on the insurer's *whole* portfolio (including the 'small contracts') is suitable at all. After all, such a limitation of risk-adjusted pricing can be significant. Thus, a pragmatic solution might be recommended.

The approach of Subsection 6.2.1 may be used in the same way, if the change in the credibility weight α_i from zero to L is adjusted when a contract passes the volume limit V^L. Possibly after a careful choice of a sufficient number of risk groups in order to allow for enough risk differentiation, it is then suggested to control the behaviour of α_i by determination of the credibility coefficient as follows:

$$\kappa = \frac{\sigma^2}{\tau^2} := V^L \cdot \frac{1-L}{L} \cdot \sum_{j \in T} f_j \ .$$

Henceforth, no estimation of structural parameters is necessary for the risk level of an individual contract in relation to its risk group, and the same results as within the preceding subsection hold true.

6.3 Separate treatment of large contracts

Typically, an insurer prefers to sell large volume contracts than low volume ones. Possible reasons for this are: higher volume typically relates to higher profits, maintenance and administration are easier, risk data is more significant, insurance coverage might be higher, and expenses per insured are considerably lower (in fact, to avoid premiums being too high for small contracts, there might be a cross-subsidy of that part of the premium to cover expenses within the portfolio). Though sometimes carefully monitored by Insurance Regulators, an insurer might want to assess, whether lower risk premiums are justified for large volume insurance contracts to be more competitive. This subsection discusses some ideas, but does not provide a generic solution to the topic.

Indeed, there might be reasonable arguments for a better average risk behaviour in large companies when compared with small ones, e.g. for the disability risk in group life insurance business:

- Large companies might have structures and personnel to monitor employees during work incapacity. After periods of care, recovery of the individual is more probable. In addition, a large company might be more likely to be able to offer a gentle return into daily business in terms of a reduced workload, or be able to make available special facilities for disabled employees.

- If there are many departments available, it might be possible to offer occupational retraining to employees due to natural fluctuation in personnel. Therefore, both recovery from disability, and preventative measures, seem more probable.

- Large companies might be less directly exposed to volatility in the labour market. Thus a more stable and reliable environment is offered to staff, where negative shocks can be borne.

The introductory report, [T11], addressed the two-layer approach to combine two stages of risk levels in one single risk adjustment: once the risk group is judged in relation to the insurer's whole portfolio, and secondly the individual contract is judged in relation to the corresponding risk group. As a possible approach to allow for further diversification in terms of the volume of a contract, one might consider introducing a *third layer* (see Figure 10). This might either be done according to the scheme of the present two layers, or by using *hierarchical credibility theory*.[18]

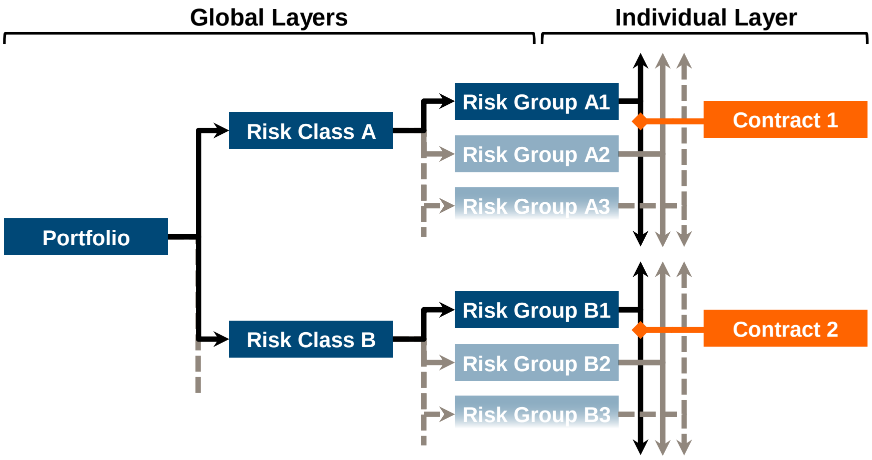


Figure 10: Three-layer model

However, this approach requires careful consideration, as for example:

- Should the upper global layer, the *risk classes*, consider the volume of contracts within the portfolio, being further divided into *risk groups* according to, for instance, industry sectors? Or would the other way round ensure a broader differentiation of risk-judgments, as some branches of the hierarchical tree in Figure 10 might provide too few observation data? Moreover, large contracts might, on average, even be a worse risk than small ones in some industry sectors. However, the sequential arrangement significantly affects the model philosophy.

- The separation of the portfolio into risk classes according to a contract's volume presupposes the definition of fixed volume limits to allot a contract to either of the risk classes. How should the limits be chosen, and what happens, if a contract passes a volume limit from one year to another?

- At which stage in the hierarchical tree should the structural parameters, σ^2, τ^2 and f_j $(j = 1, \dots, n)$, be estimated? A serious problem arises if a contract is re-allotted within the global layers. For instance, a contract of the construction industry grows in volume and changes from A1 to B1, or a research company focuses further on the production of a certain product and changes from A2 to A3 (due to the different industry sector). The *risk structure* of the former global entity (measured by σ^2 and τ^2) might considerably differ from that of the latter. But this structure has a major impact on the weight of the individual risk experience in the final risk-judgment, and thus it might happen that the 'same' contract will pay a *larger* risk premium after the change into a global entity with a *better* average risk than before.

  One might therefore consider estimating σ^2 and τ^2 on a higher hierarchical level and using the same parameters for all lower layers. However, the frequencies f_j do not allow for such an approach.

[18] The interested reader is referred to [BG05] for a detailed introduction to hierarchical credibility theory.

Alternatively, one might consider using just two layers, but letting some of the risk groups represent ‘large contracts’. Regardless of whether such an idea is reasonable or not, the reader should note that the above issues arise. In addition, independence of the risk groups would be even more questionable.

As a conclusion, the incorporation of the volume of a contract as a *risk measure* (rather than as a measure for the credibility of an observation by the risk measure) within a sound credibility theory approach for group life pricing could appear problematic, if possible at all. Further research in this area falls outside the scope of this paper.

6.4 Outlier observations

This last subsection discusses the treatment of outlier observations within the observation period in the context of a credibility model. Such extraordinary large claims (be it a single, large claim amount, or an accumulation of claims) might affect the risk judgment to a considerable extent, though they might not exhibit much statistical significance in terms of the risk characteristics. Moreover, the possible occurrence of large claims gives rise to, on average, smaller credibility weights for small and medium volume contracts, due to increased volatility estimator. Consequently, risk adjustments vary little among those contracts, assuming there was no outlier observation.

Many approaches providing a solution to this issue can be found in literature. Bühlmann and Gisler, [BG05], give a good overview about the topic: One of the first ideas was to *apply credibility to robust statistics*, where basically the observed individual arithmetic mean of claims is replaced by a more robust estimator, such as the median, for instance. Such an approach might be feasible in the context of Section 2, where the robust estimator could be applied within the standardised observed claim frequencies. Gisler and Reinmann have further developed the application of robust statistics in [GR93], where a *volume-dependent truncation level* is individually, and ‘automatically’, applied to the observation data.

One might also think of the multidimensional model in Section 3 as a possible solution to the issue, as the impact of observations from the past on the resulting risk adjustment is limited. Although the tool presented in Section 4 also allows for a reduced weight of the individual risk experience (and thus, of the outlier observation), this has to be seen as a manual intervention and does not work on an ‘unmonitored model’ basis.

Furthermore, Bühlmann and Gisler suggest *semi-linear credibility with a truncation transformation* on the observation data. The idea is to cut off observed claims amounts exceeding a *global threshold* and assume additional structure in the claims numbers, when observation data is used to compute the credibility estimator. Nevertheless, the excluded parts are taken into account within the resulting credibility premium in a statistically sound manner.

However, the use of a *global* threshold, valid for a whole sub-portfolio rather than for an individual contract only, could be seen as a drawback. A workaround would be to divide the insurer’s portfolio into volume classes, where the threshold is chosen for each class individually. By an argument similar to the idea in Subsection 4.2.3, one could then chose the threshold for a certain class in such a way, that the maximum relative impact of a single outlier observation on the final risk adjustment is limited to a pre-defined maximum level, say, 20%. In the end, such a construction allows for even more flexibility and control than the previously mentioned.

Finally, it should be emphasized that the treatment of an outlier claims *amount* seems reasonable in most cases (as it can be seen as *random* event), whereas for an extraordinary *accumulation of claims* it might a priori not be clear whether truncation, for instance, is justified (as its cause could be *systematic*). Therefore, model adjustments with respect to outlier observations should be carefully chosen according to the specific circumstances of an insurer.

7. Some final thoughts

In Sections 2 to 6, nine different topics are addressed regarding typical issues an insurer might encounter in daily business. Each of the approaches discussed should be seen as an extension, or a further development, of the model and the techniques presented in the introductory report, [T11].

The risks under consideration throughout this paper have been disability and mortality within group life business. Nevertheless, the presented ideas apply generally to credibility theory and are not limited to group life insurance and the specific model in [T11]. Furthermore, if an insurer is exposed to more than one of the outlined issues, the corresponding model modifications can usually be merged into a single model.

As a preliminary stage to *evolutionary credibility theory*, Section 5 provided insight regarding the fact that the risk characteristics of an entity should rather be seen as a stochastic process than as fixed quantity. Indeed, there are many factors continuously influencing the risk characteristics of a group of insured lives as a whole, and some of these are unknown. This was formerly taken into account by introducing a 'volatility margin' within the model assumptions, which is still based on the assumption that, *on average*, risk characteristics remain constant. Certainly, a more sophisticated approach would be to use an evolutionary credibility model. There, the observations are the same, that is, R_{ij} is the observed claims frequency of the individual entity i during the year j. But the individual risk profile is allowed to *change stochastically with the passage of time*. The resulting credibility estimator is thus also a real-valued stochastic process. Moreover, if independence between the risk characteristics does not hold, e.g. if changes in the law or the economic circumstances affect a group life sub-portfolio simultaneously, we find ourselves in *multidimensional evolutionary credibility theory*.

Those models are much more complicated than the classical ones. Not only does one usually have to perform recursive calculations, e.g. using the so-called *Kalman Filter*, the appropriate choice of model assumptions and its construction might also be complex. Finally, estimation of the structural parameters needed, given a certain insurer's portfolio, can be a challenging task.[19]

In the end, it is sometimes questionable – and should be decided based upon a certain insurer's situation, its portfolio, products, and the corresponding circumstances – whether a more sophisticated pricing model leads to a better result in actual business. The *nature of those questions is complex* and therefore requires careful thought.

[19] Bühlmann and Gisler present a thorough discussion about (multidimensional) evolutionary credibility theory in [BG05].

Appendix

CALCULATIONS SUPPORTING RESULTS FROM SECTION 4

Using the notation from Section 4, the credibility estimator, φ_i, of the form

$$\varphi_i = c \cdot R_i + (1-c) \cdot R\,,$$

where $c \in \mathbb{R}$ is to be computed, has minimum expected squared error with respect to the Bayes estimator, $\varphi(\Theta_i)$. By construction, $E[\varphi(\Theta_i)] = E[\varphi_i] = E[R_i] = R$. The reader is referred to [BG05] for a detailed introduction to credibility theory, the Bühlmann and Straub model assumptions, and the derivation of the credibility estimator.

$$E\left[\left(\varphi_i - \varphi(\Theta_i)\right)^2\right] = \min_{c\in\mathbb{R}} E\left[\left(c \cdot (R_i - R) - (\varphi(\Theta_i) - R)\right)^2\right]$$

$$\begin{aligned}
0 &= \frac{1}{2}\frac{d}{dc} E\left[\left(c \cdot (R_i - R) - (\varphi(\Theta_i) - R)\right)^2\right] \\
&= E\left[(R_i - R) \cdot \left(c \cdot (R_i - R) - (\varphi(\Theta_i) - R)\right)\right] = c \cdot Var(R_i) - Cov\left(R_i, \varphi(\Theta_i)\right) \\
&= c \cdot \{E[Var(R_i|\Theta_i)] + Var(E[R_i|\Theta_i])\} - \{E[Cov(R_i, \varphi(\Theta_i)|\Theta_i)] + Cov(E[R_i|\Theta_i], E[\varphi(\Theta_i)|\Theta_i])\} \\
&= c \cdot \left\{\sum_{j=1}^{n} \frac{w_{ij}^2}{w_{i*}^2} E\left[Var\left(R_{ij}|\Theta_i\right)\right] + 0 + \tau^2\right\} - \{0 + \tau^2\} = c \cdot \left\{\frac{\sigma^2}{w_{i*}} + \upsilon_i \cdot \sum_{j=1}^{n} \frac{w_{ij}^2}{w_{i*}^2} + \tau^2\right\} - \tau^2
\end{aligned}$$

$$c = \frac{w_{i*}}{w_{i*} + \dfrac{\sigma^2 + \delta_i}{\tau^2}}\,, \quad \text{where} \quad \delta_i = \frac{\nu_i}{w_{i*}} \sum_{j=1}^{n} w_{ij}^2$$

CALCULATIONS SUPPORTING RESULTS FROM SECTION 5

The reader is referred to [BG05] for a detailed introduction to credibility theory and the Bühlmann and Straub model assumptions. We use the notation from Section 5.

Estimation of σ^2 and υ:

$$E[S_i] = E\left[E[S_i|\Theta_i]\right] = E\left[\frac{1}{n-1}\sum_{j=1}^{n} w_{ij} \cdot \left\{Var\left(R_{ij}|\Theta_i\right) - 2 \cdot Cov\left(R_{ij}, R_i|\Theta_i\right) + Var\left(R_i|\Theta_i\right)\right\}\right]$$

$$\begin{aligned}
&= E\left[\frac{1}{n-1}\sum_{j=1}^{n} w_{ij} \cdot \left\{\frac{\sigma^2(\Theta_i)}{w_{ij}} + \upsilon - 2 \cdot \frac{w_{ij}}{w_{i*}} Var\left(R_{ij}|\Theta_i\right) + \sum_{k=1}^{n} \frac{w_{ik}^2}{w_{i*}^2} Var\left(R_{ik}|\Theta_i\right)\right\}\right] \\
&= E\left[\frac{1}{n-1} \cdot \left\{n \cdot \sigma^2(\Theta_i) + w_{i*}\upsilon - 2 \cdot \left(\frac{w_{i*}}{w_{i*}}\sigma^2(\Theta_i) + \upsilon \sum_{j=1}^{n} \frac{w_{ij}^2}{w_{i*}}\right) + \sum_{j=1}^{n} w_{ij} \cdot \left(\frac{1}{w_{i*}}\sigma^2(\Theta_i) + \upsilon \sum_{k=1}^{n} \frac{w_{ik}^2}{w_{i*}^2}\right)\right\}\right] \\
&= E\left[\frac{1}{n-1} \cdot \left\{(n-1) \cdot \sigma^2(\Theta_i) + \upsilon \cdot \left(w_{i*} + \sum_{j=1}^{n} \frac{w_{ij}^2}{w_{i*}}\right)\right\}\right] = \sigma^2 + \frac{w_{i*}}{n} \cdot \frac{n}{n-1} \cdot \sum_{j=1}^{n} \frac{w_{ij}}{w_{i*}}\left(1 - \frac{w_{ij}}{w_{i*}}\right) \cdot \upsilon
\end{aligned}$$

Estimation of τ^2:

$$Var(R_i) = E[Var(R_i|\Theta_i)] + Var(E[R_i]|\Theta_i) = E\left[\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{i*}^2}Var\big(R_{ij}|\Theta_i\big) + 0\right] + Var\left(\sum_{j=1}^{n}\frac{w_{ij}}{w_{i*}}E\big[R_{ij}|\Theta_i\big]\right)$$

$$= \sum_{j=1}^{n}\frac{w_{ij}^2}{w_{i*}^2}\left(\frac{\sigma^2}{w_{ij}} + \upsilon\right) + Var\big(\varphi(\Theta_i)\big) = \frac{\sigma^2}{w_{i*}} + \upsilon\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{i*}^2} + \tau^2$$

$$Var(\bar{R}) = \sum_{i=1}^{I}\frac{w_{i*}^2}{w_{**}^2}Var(R_i) + 0 = \frac{\sigma^2}{w_{**}} + \upsilon\sum_{i=1}^{I}\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{**}^2} + \tau^2\sum_{i=1}^{I}\frac{w_{i*}^2}{w_{**}^2}$$

$$Cov(R_i,\bar{R}) = \sum_{k=1}^{I}\frac{w_{k*}}{w_{**}}Cov(R_i,R_k) = \frac{w_{i*}}{w_{**}}Var(R_i) = \frac{\sigma^2}{w_{**}} + \upsilon\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{**}w_{i*}} + \tau^2\frac{w_{i*}}{w_{**}}$$

$$E[T] = \frac{I}{I-1}\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}E\left[\big((R_i - R) - (\bar{R} - R)\big)^2\right] = \frac{I}{I-1}\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}\{Var(R_i) - 2\cdot Cov(R_i,\bar{R}) + Var(\bar{R})\}$$

$$= \frac{I}{I-1}\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}\left\{\frac{\sigma^2}{w_{i*}} - \frac{\sigma^2}{w_{**}} + \upsilon\cdot\left[\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{i*}^2} - 2\sum_{j=1}^{n}\frac{w_{ij}^2}{w_{**}w_{i*}} + \sum_{k=1}^{I}\sum_{j=1}^{n}\frac{w_{kj}^2}{w_{**}^2}\right] + \tau^2\cdot\left[1 - 2\frac{w_{i*}}{w_{**}} + \sum_{k=1}^{I}\frac{w_{k*}^2}{w_{**}^2}\right]\right\}$$

$$= \frac{I\cdot\sigma^2}{w_{**}} + \frac{I\cdot\upsilon}{I-1}\sum_{i=1}^{I}\sum_{j=1}^{n}\frac{w_{ij}}{w_{**}}\left(\frac{w_{ij}}{w_{i*}} - \frac{w_{ij}}{w_{**}}\right) + \frac{I\cdot\tau^2}{I-1}\sum_{i=1}^{I}\frac{w_{i*}}{w_{**}}\left(1 - \frac{w_{i*}}{w_{**}}\right)$$

ACKNOWLEDGEMENTS

Special gratitude is owed to Prof Dr Alois Gisler of the Swiss Federal Institute of Technology (ETH) Zurich. His revealing insight, suggestions for new ideas, and participation in vivid discussions provided a distinguished contribution to this report.

Further, the author would like to thank the following people for their valuable feedback and input for this paper:

Nick Kinrade

Scott Mitchell

SOURCES

[GR93] Gisler A., Reinhard P., 'Robust Credibility', ASTIN Bulletin Article, Vol. 23, No. 1, 1993.

[BG05] Bühlmann H., Gisler A., 'A Course in Credibility Theory and its Applications', published by Springer, 2005.

[F05] Fringeli M., 'Credibility für Probleme mit räumlicher Abhängigkeit', Master Thesis at the Swiss Federal Institute of Technology (ETH) Zurich, 2005.

[T08] Tschupp M., 'Credibility in der Kollektiv-Lebensversicherung', Master Thesis at the Swiss Federal Institute of Technology (ETH) Zurich, February 2008, http://math.ethz.ch/Finance/waltersaxer.

[M11] Mancini A., 'Anwendung der Kredibilitätstheorie zur Schätzung von Sterbewahrscheinlichkeiten', Master Thesis at the Swiss Federal Institute of Technology (ETH) Zurich, March 2011.

[T11] Tschupp M., 'Application of Credibility Theory to Group Life Pricing; An Introduction', Milliman research report, August 2011, http://ch.milliman.com/en/perspective/market-research/application-of-credibility-theory.php.

CONTACT DETAILS

For further details relating to any aspect of this report, please contact the author, or your local Milliman representative:

Manuel Tschupp
Milliman AG
Lavaterstrasse 65
CH-8002 Zurich
Switzerland

Tel: +41 44 287 9076
manuel.tschupp@milliman.com

ABOUT MILLIMAN

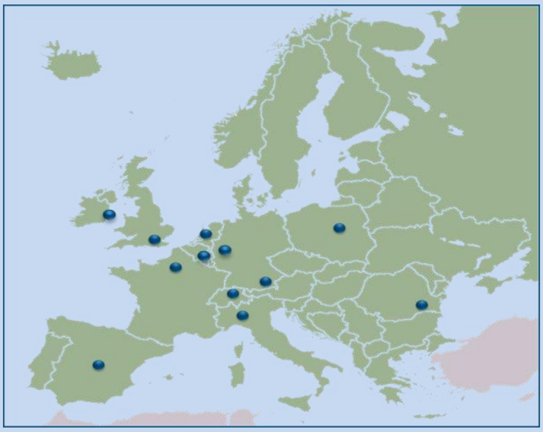
Milliman is among the world's largest independent actuarial and consulting firms. Founded in 1947, the company currently has 56 offices in key locations worldwide employing more than 2,500 people.

www.milliman.com

MILLIMAN IN EUROPE

Milliman maintains a strong and growing presence in Europe with 250 professional consultants serving clients from offices in Amsterdam, Brussels, Bucharest, Dublin, Dusseldorf, London, Madrid, Milan, Munich, Paris, Warsaw, and Zurich.

www.milliman.ch



CONTACT

For more information, please contact your local Milliman representative, or the author:

Manuel Tschupp
manuel.tschupp@milliman.com
+41 44 287 9076