Practical challenges and possible solutions for life insurers relating to the Delta-Gamma approach under the Swiss Solvency Test

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As part of our series of papers on Swiss Solvency Test ('SST'), this short paper considers the challenges involved for life insurers in implementing the 'Delta-Gamma' approach for market risk under the SST standard model, as well as various possible solutions to this problem.

INTRODUCTION

In 2011, the 5-year phase-in period of the Swiss Solvency Test ('SST') ended, and now all insurers are required to meet the SST capital requirements. The SST is a principles- and risk-based solvency regime. Swiss-based insurers can choose to adopt the so-called 'standard model' approach or, alternatively, implement an internal model, which is subject to regulatory approval.

In this paper, we focus on implementation challenges relating to the so-called 'Delta-Gamma' approach under the SST standard model, as well as possible practical solutions to these challenges.

The implementation of the Delta-Gamma approach is particularly challenging for Swiss-based life insurers. A significant portion of life insurance contracts in Switzerland (and other European insurance markets) contains guaranteed rates and profit sharing features. Under SST, such contracts require stochastic valuation in order to accurately capture the time value of financial options and guarantees. However, as we will describe below, implementing the Delta-Gamma approach in a stochastic model can result in significantly impractical run-time issues.

BACKGROUND TO THE DELTA-GAMMA APPROACH

The Delta-Gamma approach for market risk became obligatory for SST standard model 2010 year-end reporting. This followed the publication of a paper in 2007 (see [3]), which considered second-order impacts for risk factors. Prior to 2011, only first-order impacts were considered by SST, based on the 'Delta-Normal' approach. Both approaches are described below.

The Target Capital under the SST standard model is based on aggregation of risk capital for a set of prescribed risk factors. The estimated change in Risk Bearing Capital ('RTK')[1] is calculated using a multivariate approximation, where sensitivities of the Risk-Bearing Capital to the various risk factors are used to approximate the corresponding partial derivatives. The Target Capital is then evaluated using the Tail Value at Risk[2] of the distribution of the RTK at the 99% confidence level.

The Delta-Normal approach to market risk was based on a first-order approximation of the 77 risk factors over a one year time horizon. Mathematically, this is expressed via a simple first-order Taylor expansion as:

$$\Delta RTK = \sum_{n=1}^{N} \frac{\partial RTK}{\partial z_n} \Delta Z_n,$$

where:

ΔRTK is the change in RTK over a one year time period

ΔZ_n is the change in the nth risk factor over a one year time period

N is the number of market risk factors considered by the SST standard model.

[1] We use the German acronym for 'Risikotragendes Kapital', or Risk Bearing Capital.

[2] Also referred to as Tail VaR or Expected Shortfall.

The Delta-Gamma approach uses a more accurate approximation by introducing a second-order component into the evaluation of Target Capital, which can be expressed mathematically as:

$$\Delta RTK = \sum_{n=1}^{N} \frac{\partial RTK}{\partial z_n} \Delta Z_n + \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{\partial^2 RTK}{\partial z_n \partial z_m} \Delta Z_m \Delta Z_n.$$

This modification is intended to capture second order effects. Such dependencies occur typically in life insurance portfolios with guaranteed interest rate business. In general the most material differences from the previous Delta-Normal approach are expected to arise from market risks, such as interest rates.

ESTIMATING PARTIAL DERIVATIVES

As shown before, under the Delta-Normal approach, two model runs were needed for each risk factor (+/- 1% absolute change in interest rate and spread risk factors, and +/- 10% relative change for other risk factors). The changes in the RTK, ΔRTK_+ and ΔRTK_-, are then used to approximate the first-order partial derivative as follows:

$$\frac{\partial RTK}{\partial z_n} \approx \frac{\Delta RTK_+ - \Delta RTK_-}{2\Delta z_n}$$

Similarly, under Delta-Gamma, the second-order full partial derivatives can be estimated from the existing runs, as follows:

$$\frac{\partial^2 RTK}{\partial z_n^2} \approx \frac{\Delta RTK_+ + \Delta RTK_-}{(\Delta z_n)^2}$$

To estimate the second-order cross partial derivatives further runs are needed. If we consider two risk factors and $\Delta \mathrm{RTK}_{-+}$ for example represents the change in RTK resulting from the down-shift in the first risk factor and the up-shift in the second risk factor, then we can estimate the cross partial derivative as follows:

$$\frac{\partial^2 RTK}{\partial z_n z_m} \approx \frac{\Delta RTK_{++} - \Delta RTK_{+-} + \Delta RTK_{--} - \Delta RTK_{-+}}{4\Delta z_n \Delta z_m}$$

where $n \neq m$.

CHALLENGES OF IMPLEMENTATION

Under the SST standard model there are currently 77 individual market risk factors. Such a large number of risk factors has the advantage of better capturing specific risk profiles, but can have significant practical implications in terms of calculations required and interpretation of results. These practical implications have been magnified significantly following the introduction of the Delta-Gamma approach.

The first-order partial derivatives are estimated by calculating two sensitivities for each risk factor. This results in up to 154 model runs to calculate the Delta-Normal approximation for Target Capital for market risk, depending on how many risk factors a company is exposed to.

Under the Delta-Gamma approach, the number of possible runs for market risk increases drastically.

For example, for interest rate risk, which is typically the most significant risk exposure for life insurers in Switzerland, there are 338 model runs required for each currency[3]. For companies exposed to all four prescribed SST currencies (CHF, EUR, USD and GBP), this results in 1,352 model runs for interest rates, before allowing for additional runs stemming from inter-dependencies between currencies.

At this stage, the scope of calculations might already be an unmanageable task for many companies. Additional market risk factors, such as equity, property, exchange rates, and volatilities introduce further complexity and challenges for model run-time, process and validation.

For a diversified insurance group with exposure to many interrelated risk factors, the implications for model run-times can be staggering. Perhaps the only feasible solution for such companies is to adopt an internal model.

In short, the SST Delta-Gamma approach demands an unprecedented level of planning and automation in model processes. The associated operational risks, such as model or

[3] For interest rates in a single currency, there would be 2x13=26 first order sensitivity runs, plus 4x(13x13-13)/2= 312 second order sensitivity runs.

parameterisation error, should also not be underestimated. There is also a significant risk that, because companies will spend so much effort and time simply preparing calculations and model results, management potentially has little time to analyse results and focus on risk management decision-making consequences.

Because SST requires the Target Capital to be determined over a 1-year time horizon, the most 'correct' methodology is to adopt a nested stochastic approach[4]. However, nested stochastic calculations raise further practical computational issues, due to the dramatic increase in run-time of actuarial models. These computational issues are a key driver behind the development of proxy modelling techniques, which are described in the next section.

POSSIBLE PROXY MODELLING TECHNIQUES

Unless a company chooses to adopt an internal model, which can be onerous for small- or medium-sized companies to implement, life insurers have limited choices in dealing with the practical problem arising from the Delta-Gamma approach.

Given the nature of the SST standard model risk factors, a life insurer should either focus on

a. implementing acceleration techniques for the modelling process and/or run-times; and/or

b. reducing the number of model runs, for example by excluding runs which are not statistically significant.

Increased computing power, including 'cloud computing', has become an important focus area when considering faster run-times of stochastic models. However, in this paper, we consider a variety of proxy modelling techniques which are available to life insurers and specifically designed to accelerate the calculations.

- **Replicating Portfolios:**

Replicating Portfolios are conceptually appealing and have been widely promoted in Switzerland as a possible approach for determining Target Capital for market risks. Implementation of Replicating Portfolios can be considered as moving towards the development of an internal model. Nevertheless, we include it here as a possible approach for coping with the challenges of the Delta-Gamma approach. For detailed discussion of Replicating Portfolio techniques, please refer [2].

Despite the apparent advantages, it should be highlighted that Replicating Portfolio techniques are not without limitations.

While Replicating Portfolios can capture the hedgeable market risks inherent in insurance portfolios, they are less appropriate for representing non-market risks or non-hedgeable market risks. Thus different modelling techniques need to be used for such risks, resulting in potential inconsistencies across risk factors.

For certain life insurance contracts, the effectiveness of this technique can be particularly questionable and the implementation challenges can be difficult to overcome.

In particular, cashflows from participating life contracts can be challenging to replicate using tradeable (or even synthetic) assets. This difficulty is largely caused by the non-hedgeable market-related features which are inherent in life insurance contracts. For example, complex dynamic management actions and dynamic policyholder behaviour are significant drivers of liability cashflows, yet these aspects are often not adequately reflected.

Experience has highlighted uncertainty around whether calibration scenarios fully capture all possible cases, resulting in poor predictive power of the Replicating Portfolio. For substantial market movements, this can result in significant differences between the market value of the Replicating Portfolio and that of the respective liabilities.

A major implication is that the Replicating Portfolio might not capture the impact of dynamic management actions or dynamic policyholder behaviour in extreme market risk scenarios, and therefore such risks are potentially not captured in the Target Capital.

Since participating business makes up a significant portion of life insurance liabilities in Switzerland, and market-related risk is the main

[4] In comparison, the Solvency II standard formula applies stresses at the valuation date, thus avoiding complications introduced by nested stochastic.

risk affecting such contracts (including the impact of dynamic policyholder behaviour and management actions), users should approach with caution the Replicating Portfolio technique for determining market risk Target Capital for participating life contracts.

Furthermore, it is worth highlighting that the Replicating Portfolio is potentially non-unique for participating life contracts and is dependent on the underlying investment strategy. This is because the policyholder bonuses are typically dependent on book returns, which means that the cashflows and the market value of liabilities can be influenced by the investment strategy. For example, the timing of asset sales will impact the portfolio book returns and the policyholder bonuses, and therefore the cashflows. The implication is therefore that the Replicating Portfolio is dependent on the company-specific investment strategy.

The above aspect can potentially lead to circular arguments for ALM decision-making based around Replicating Portfolio analysis. For example, when ALM decisions are based on the output of risk models (as they should be), the Replicating Portfolio for these contracts would be dependent on the investment strategy, which is in turn dependent on the Target Capital implied by the Replicating Portfolio, and so on.

Additional challenges of the technique, which were also raised by FINMA (Swiss Financial Market Supervisory Authority) in [6], include the fact that the construction of Replicating Portfolios requires strong actuarial skills and that companies often have difficulties validating the outputs to justify the use of Replicating Portfolios. FINMA has already mentioned that it is planning to ask for minimal requirements for validation which most probably will force insurers to improve their existing Replicating Portfolio processes. It is worth mentioning that other European Regulators have also raised their doubts on the appropriateness of Replicating Portfolios for Solvency II calculation purposes. In a forthcoming paper we will re-visit this point and present how appropriate validations and improvements might look like.

Despite the challenges mentioned above, there are types of life contract for which improved and properly validated Replicating Portfolios might work well, and might be useful for risk assessment and decision making. However, it is also clear that there are situations when alternative techniques are perhaps more appropriate for life contracts.

- **Least Squares Monte Carlo:**

Least Squares Monte Carlo (LSMC) is a technique which enables an accurate calculation of the probability distribution forecast. As such, this approach is a strong modern technology for internal models. However, we believe that LSMC is also an interesting candidate technique in the standard model context.

LSMC does not incur any significant software licence fees and can be validated in a robust and reliable way. It is therefore an attractive modelling option from a budgetary perspective.

Under the LSMC approach, one assumes that an economic balance sheet position can be represented as a polynomial function of the risk drivers relevant for the insurer – such as interest rates, equity volatility, corporate bond default level, lapse level or mortality level. The method consists of the following four steps:

- **Step 1:** Populate the multi-dimensional state space of possible risk driver values thoroughly by using, say, 25,000 test points and calculate the corresponding balance sheet for each of these points by using just two inner scenarios emanating from each point. These results represent rough estimates.

- **Step 2:** Estimate the coefficients of the polynomial function via a simple least squares regression using a standard statistical software package.

- **Step 3:** Perform out-of-sample validation of the polynomial function. For several points of the risk driver space, a full stochastic calculation is performed and its result is compared to the corresponding value of the polynomial function. The precision of the method can be assessed by calculation of confidence intervals and standard errors.

- **Step 4:** Evaluate the polynomial function by using a set of, say, 100,000 one-year real world scenarios featuring the desired dependencies between the relevant risk drivers. Obtain the probability distribution forecast from the evaluation results.

In this manner, the full probability distribution can be accurately calculated, yielding relevant information for risk management purposes. In particular, the inter-play between different risk types can be analysed and interpreted economically by considering two-dimensional cross-sections of the risk driver space – for example, the cross-section in the dimensions “interest rate level” and “lapse level”. Furthermore, knowledge of the probability distribution provides such risk measures as Value at Risk and Tail Value at Risk.

From the process perspective, LSMC offers a number of important benefits. In particular, quick updates, with a high degree of accuracy, of the company’s risk position are possible. Risk managers can analyse a complete risk landscape, in which all the risks including insurance risks are treated consistently – this is an important advantage over the replicating portfolio method. Furthermore the single steps in the LSMC process can be automated to a high degree.

For more detailed discussion of the LSMC technique, please refer [8].

- **Cluster Modelling**

Cluster Modelling techniques, adopted from areas of social science and other applications, have been successfully implemented in the context of life insurance modelling. These techniques have been developed specifically to support more efficient stochastic model runs, including nested stochastic.

The overarching idea behind Cluster Modelling is that individual policies are treated as objects in a multi-dimensional space and ‘similar’ policies are mapped together in clusters.

Cluster Modelling can be used to compress policy liability data, economic scenarios or asset data, either for stochastic valuation or possibly as part of the calibration process for the Replicating Portfolio. Clustering analysis techniques enable users to efficiently transform millions of policies into just a few thousand, or even a few hundred, model points. Results of complex cashflow projections over numerous economic scenarios can therefore be obtained reliably and quickly.

In this sense, some of the advantages of Replicating Portfolios are reproduced. However, Cluster Modelling has the additional advantage that fast calculation of sensitivity results to non-market and non-hedgeable risk factors can also be produced, including dynamic policyholder behaviour or dynamic management actions.

For more detailed discussion of Cluster Modelling techniques, please refer [7].

- **Dimension reduction techniques**

Other alternative techniques have been recently proposed, such as dimension reduction for interest rate risk. Such techniques are outside the scope of this short paper, and we refer interested readers to [1] for further reading.

- **Other ‘proxy’ techniques**

Other techniques, such as Curve Fitting and ‘Nested Stochastic Accelerator’ techniques are available for determining Target Capital. However, these techniques are better suited for an internal model, rather than the SST standard model, and are therefore outside the scope of this paper.

CONCLUSIONS

The Delta-Gamma approach to market risk under the SST standard model introduces an unprecedented level of computational challenges for life insurers.

A variety of alternative options are available for life companies to consider when performing the SST calculations. Different modelling techniques will be appropriate in different situations and potentially for different products. It is important to assess which solutions are best-suited to a life insurer’s specific situation and risk profile.

For example, Replicating Portfolio techniques can be useful in certain circumstances. However, the Replicating Portfolio may prove difficult to calibrate for participating life contracts, and may not capture appropriately the risk profile of important non-hedgeable factors such as dynamic policyholder behaviour or dynamic management actions.

In these circumstances, other techniques such as LSMC, Cluster Modelling or nested stochastic might be more appropriate.

Even with the availability of such proxy modelling techniques, as well as more advanced technology, the practical issues arising from the introduction of the Delta-Gamma approach remain significant, as compared to other similar frameworks (e.g. Solvency II). As a result, life companies may view the internal model as a more feasible option for SST reporting.

HOW MILLIMAN CAN HELP

As one of the world's largest actuarial and consulting firms, Milliman has supported numerous life insurance clients across Europe on Solvency II and SST model implementations.

Our consultants have experience with the practical challenges discussed in this paper and developed new approaches to Replicating Portfolios, LSMC, Cluster Modelling and nested stochastic techniques, specifically designed to deal with the problems mentioned in this article.

For more information, please contact your local Milliman consultant or one of the authors.

REFERENCES & FURTHER READING

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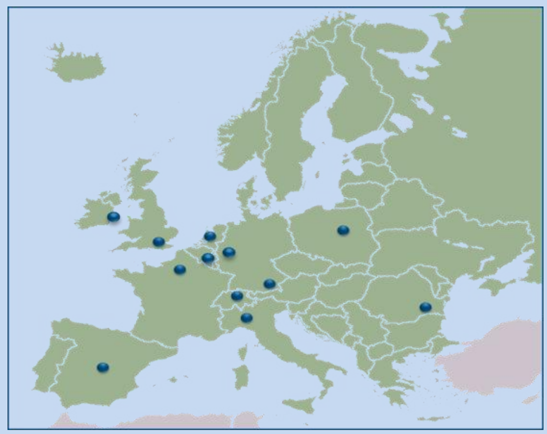

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