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The three important ingredients in an MVO model are the alpha vector representing expected returns, the risk model that is used to measure the variance of the portfolio, and a set of constraints representing the portfolio managers' mandates and choices. In the traditional quantitative investment process, these three inputs are usually developed independently of each other, without much regard to the interaction between them. As a result, the optimal portfolio generated by the traditional approach may not consistently represent the views of the portfolio manager that are expressed in the expected returns. This paper uses the consistent investment approach outlined in Stubbs (2013) to propose a portfolio construction process that takes into account the interaction between all the components of the MVO model in order to yield a more transparent process that translates superior returns into outperforming portfolios. The consistent investment approach generates the optimal portfolios in three steps: (a) by converting the individual signals used by the portfolio manager into factor mimicking portfolios (FMPs); (b) by linearly combining the FMPs into a target portfolio; and (c) by finding the optimal portfolio that is similar to the target portfolio but that also satisfies all the additional constraints imposed by the portfolio manager. We demonstrate the effectiveness of the consistent investment approach over the traditional investment approach by comparing them on a practical example.

Alpha Construction in a Consistent Investment Process

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1 Introduction

Markowitz (1952, 1991) developed the mean variance optimization (MVO) model that is widely used in portfolio management. The MVO model achieves an optimal tradeoff between risk and return by solving a quadratic optimization problem that is based on a quadratic utility function that considers the first two moments of asset returns, namely the mean and the variance, to measure the return and the risk of the portfolio, respectively. In addition to trading off risk and return, the MVO model has been extended to include a set of constraints that model additional business requirements imposed by asset owners, regulators, risk managers, and trading desks alike. Additional constraints are also frequently used by portfolio managers to implement investment insights or in order to overcome some of the shortcomings of the MVO model itself.

In this paper we concentrate on studying how these three main ingredients of the MVO model interact with the optimizer to produce optimal portfolios. These inputs to the MVO model include: the alpha vector, representing the expected returns; the risk model, that is used to measure the variance of the portfolio, and a set of constraints. In the traditional quantitative investment process, these three inputs to the MVO model may be developed independently of each other, without much regard to the interaction between them.

The main challenge with this *independent* approach to the generation of the optimal portfolio is that this portfolio may not consistently represent the views of the portfolio manager that are expressed in the expected returns. This paper uses the consistent investment process outlined in Stubbs (2013); that builds on earlier work in Grinold and Kahn (2000), Black and Litterman (1990, 1992), and Sefton et al. (2004) to propose a portfolio construction process that takes into account the interaction between all the components of the MVO model to yield a more transparent process that translates superior expected returns into outperforming portfolios.

The consistent investment approach, which is described in more detail in Section 3, generates the optimal portfolios in three steps: (a) by converting the signals the portfolio manager uses to construct the expected returns into individual portfolios, called factor mimicking portfolios (FMP); (b) by linearly combining the factor mimicking portfolios into a target portfolio, which represents an *idealized* portfolio that optimally combines the signals; and (c) by finding the optimal portfolio as a portfolio that is similar to the target portfolio, but that also satisfies all the additional constraints imposed by the portfolio manager. In contrast to the consistent investment approach, the traditional MVO framework solves a single portfolio optimization problem including the expected returns, the risk model, and all the additional constraints.

The paper is organized as follows: Section 2 provides a brief overview of the MVO model and discusses various challenges. Section 3 describes the main steps of the consistent investment process in detail. The consistent investment process is illustrated on a practical example in Section 4. Section 5 presents some of our conclusions. We discuss some technical details in Appendix A.

2 Mean Variance Optimization

There are three main ingredients to an MVO model: the α vector representing the expected returns, the risk model Q, and the constraint set $\mathcal{C}$ that defines the strategy employed by the portfolio manager. Consider a portfolio with an investment universe of n assets. Let h_i denote the weight (proportion of total funds) invested in the ith asset. Let α_i denote the portfolio managers' estimate of the expected return for the ith asset.

We define a *signal* or *factor* as a vector of asset characteristics that explains the cross-section of returns. Throughout this paper, we will differentiate between two types of factors: *alpha* factors that exhibit a long term predictable return trend, and *risk* factors that do not exhibit such a predictable trend. Examples of *alpha* factors include value, momentum, and growth; while examples of *risk* factors include industries and countries.

Let X_A and X_R denote the asset exposures to the alpha and risk factors, respectively. [1] Let m be the number of alpha factors and let k be the number of risk factors. Assume that the portfolio manager constructs the overall alpha factor from m different factors in X_A. The risk model is given by

$$Q = X\Omega X^T + \Delta^2 \tag{1}$$

where

$$X = [X_A, \;\; X_R]$$

[1] The cross sectional regression model has the form

$$r = X_A f_A + X_R f_R + r_{res}$$

where r, f_A, f_R represent the excess (over the risk free rate) asset, alpha factor, risk factor, returns respectively. So, the exposures X_A and X_R actually represent the *betas* or sensitivities of the assets to the alpha and the risk factors in the model. The term *alpha* comes from the fact that the residual return r_{res} can be further decomposed as

$$r_{res} = \alpha + \epsilon$$

where α is the average residual return and ϵ is a mean zero random component of residual return. The average residual return term is termed *alpha* since the factors X_A in the regression model are known to have a positive *risk premium*.

is the combined matrix of factor exposures,

$$\Omega = \begin{bmatrix} \Omega_A & \Omega_{AR} \\ \Omega_{RA} & \Omega_R \end{bmatrix}$$

is the factor covariance matrix, and Δ^2 is a diagonal matrix of specific variances. The overall MVO model is given by

$$\max_{h \in \mathcal{C}} \quad \alpha^T h - \frac{\lambda}{2} h^T Q h, \tag{2}$$

where $\lambda > 0$ is an appropriate risk aversion parameter and $\mathcal{C}$ contains the different constraints in the MVO model.

We conclude this section by introducing some concepts that will be used later in the paper. The information ratio (IR) of a portfolio is a measure of the risk-adjusted return of the portfolio, namely, the expected active return of the portfolio divided by the standard deviation of the expected active return. Given a portfolio h^* and a risk model Q, the *implied alpha* is the alpha vector that would yield h^* as the solution to the MVO problem without any constraints. Let $\tilde{\alpha}$ denote the implied alpha of the portfolio. It is easy to demonstrate that

$$\tilde{\alpha} = Q h^*. \tag{3}$$

Clarke et al. (2006) developed the Transfer Coefficient (TC), which is used to measure the efficiency with which the alpha signal is transferred to the optimal portfolio. The transfer coefficient (TC) is given by

$$\begin{aligned} \text{TC} &= \frac{\alpha^T h}{\sqrt{\alpha^T Q^{-1} \alpha} \sqrt{h^T Q h}}, \\ &= \text{corr}(Q^{-1/2}\alpha, Q^{1/2}h). \end{aligned} \tag{4}$$

The TC can be interpreted as the correlation between the risk-adjusted alpha, $Q^{-1/2}\alpha$, and the risk-weighted portfolio, $Q^{1/2}h$. [2] The TC also represents the correlation between the risk-weighted final portfolio and the risk-weighted unconstrained MVO portfolio (where there are no constraints). In a sense, the TC measures how close the final portfolio is to the unconstrained MVO portfolio. Constraints such as turnover and asset bounds in a realistic strategy lower the TC from its ideal value of 1.

3 The Consistent Investment Process

There are three main steps in the consistent investment process proposed in Stubbs (2013). We first provide a brief description of these steps and then discuss them in detail in Sections 3.1, 3.2, and 3.3.

[2] TC represents the correlation between $Q^{-1/2}\alpha$ and $Q^{1/2}h$ only if the mean of α and h are zero.

1. **Transform each alpha signal into factor mimicking portfolios:** Each alpha factor (signal) is transformed into a factor mimicking portfolio (FMP). The FMP is generated by solving an optimization problem which minimizes some measure of the portfolio risk while controlling the exposure of the FMP to other risk and alpha factors. The goal of this optimization problem is to have the FMP replicate the alpha factor returns, while neutralizing other factor exposures. Because we use an optimization model to construct the FMP, we can add constraints to this problem in order to generate a more realistic FMP. We will illustrate the benefits of adding constraints to this optimization problem in Section 4.

2. **Combine the FMPs into a target portfolio:** The second phase of the consistent investment process consists on the combination of the individual FMPs into a portfolio called the *target portfolio*. We propose to construct this portfolio through the solution of an optimization problem that trades off the risk and expected return of the FMPs. Once again, the procedure is designed to be highly flexible and allows for the inclusion of constraints in order to generate a more realistic target portfolio.

3. **Solve the final portfolio construction problem**: While the target portfolio generated in the Step 2 is the portfolio that the PM ideally would like to hold, it may not be investable and/or violate some of the other constraints that are part of the PM's mandates. The goal of this step is to generate a realistic portfolio that is as *similar* as possible to the target portfolio while satisfying all of the additional practical considerations and constraints. We solve this problem by using an MVO model once again with the following characteristics: (a) The expected returns vector is the implied alpha of the target portfolio, (b) The risk model is the matrix Q, and (c) the constraints include the relevant implementation constraints and any additional constraints that are part of the PM's mandate.

3.1 Transforming each alpha signal into factor mimicking portfolios

A factor mimicking portfolio (FMP) is a long-short, dollar-neutral portfolio that *represents* a factor. Fama and French (1992) describe simple procedures based on fractile analysis to construct such portfolios, which are also called *Fama-French portfolios* or *FF* portfolios. Although their approach to the generation of FMPs is heuristic in nature, Fama and French (1992) also neutralize other factor exposures when generating FF portfolios. For example, Fama and French (1992) neutralize their size (SMB) factor to the momentum and book attributes. Practitioners use similar techniques to neutralize their alpha signals to industry and country factors. For example, Asness (1997) neutralizes value (book to price) to industries by subtracting the market-cap weighted industry book to price average from each assets' (within that industry) book to price value.

Let m be the number of alpha signals considered by the portfolio manager. These alpha signals are represented as columns of the factor exposure matrix X_A in the risk model. The

FMP associated with the jth signal is the solution, h^j, to

$$\begin{array}{lrcl} \min & h^T W h, & & \\ \text{s.t.} & X_R^T h & = & 0, \\ & X_A^T h & = & e_j, \\ & h & \in & \bar{\mathcal{C}} \end{array} \tag{5}$$

where W is an appropriate weighting matrix, e_j is the vector with 1 in the jth position and zeros elsewhere, and $\bar{\mathcal{C}}$ contains a set of additional constraints. We will define a *pure FMP* as a dollar-neutral portfolio that has minimum total risk, has unit exposure to the alpha factor, and is neutral (zero exposure) to the other alpha factors as well as all the risk factors. For a pure FMP, the set $\bar{\mathcal{C}}$ is empty.

We can build an FMP that is not *pure* through alternative formulations of (5) along the following lines:

1. **Choosing the set of *neutral* factors:** A pure FMP is neutral to all the other factors in the risk model. Alternatively, we can impose that the FMP be neutral to only some of the risk factors in X_R and X_A. We show in the technical appendix that neutralizing for the industry factors in (5) is equivalent to some of the common heuristic industry purification schemes used in practice. The following are some alternatives we consider in the solution to (5):

   (a) The final portfolio may be required to have null or very small (active) exposure to some of the factors in the risk model. In this case, it may be beneficial to make each FMP neutral to such factors.

   (b) A particular factor may negatively contribute to the return of the final portfolio. In this case, it is better to neutralize the exposure to that factor in the generation of the FMP.

2. **Choosing the weighting matrix W:** We mention three popular choices:

   (a) W is the identity matrix, which orients the FMP towards an equal-weighted quantile spread portfolio.

   (b) $W = M^{-1}$ where M is a diagonal matrix whose entries are the asset market capitalizations. This orients the FMP towards a market-cap weighted quantile spread portfolio, where assets with a larger market capitalization are preferred.

   (c) $W = Q$, where Q is the risk model used in the portfolio construction process. In the case of pure FMPs, the covariance matrix $W = Q$ in (5) can be replaced with its diagonal specific variance component $W = \Delta^2$. On the other hand, if we do not neutralize exposures to all other factors, the resulting FMP may be unintuitive because of the correlations in Q. For example, a value FMP constructed in this way could well take large negative exposures to assets with high book to price values.

3. **Incorporating additional constraints:** Constraints from the final portfolio construction problem can also be added to (5). We mention two examples below:

   (a) The exposure of the FMP to some additional factors can be controlled if the exposure of the final portfolio to that particular factor may have a negative impact on the return of the final portfolio.

   (b) If signals have predictive abilities which are dependent on the investment horizon, we may require that the different FMPs have exposures which are related to the investment horizon of those signals. For example, Qian et al. (2007) and Gerard et al. (2012)) introduce the concept of *horizon IC* to measure the strength and the persistence of the alpha signal.

3.2 Combining factor mimicking portfolios into a target portfolio

The second phase of the consistent investment process linearly combines the FMPs of the m different alpha signals h^j into the target portfolio. Let w_j, $j = 1, \ldots, m$ be a given set of weights. We call

$$h^{tp} = \sum_{j=1}^{m} w_j h^j \tag{6}$$

the target portfolio. The weights are determined by optimally trading off the risk and the return of the different FMPs in an MVO framework.

Given a time series of returns for an FMP for each alpha signal, there are several ways to estimate the return $E[f^i]$ for each FMP and the covariance matrix Θ across all FMPs. We describe our choice below:

1. $E[f^i] = \frac{1}{T}\sum_t (r^t)^T(h^{it})$, $i = 1, \ldots, m$, where T is the total number of time periods; and r^t and h^{it} denote the time series of realized asset returns and FMP holdings, respectively.

2. Let Q denote the custom risk model that is used in portfolio construction. We have

$$\Theta_{ij} = (h^i)^T Q(h^j), \ \ i, j = 1, \ldots, m.$$

   Note that

$$(h^{tp})^T Q h^{tp} = w^T \Theta w.$$

   The optimization problem that generates the target portfolio can be written as

$$\max_w \sum_{i=1}^{m} E[f^i] w_i - \lambda w^T \Theta w \tag{7}$$

where $\lambda > 0$ is an appropriate risk threshold. We prefer to use the following equivalent formulation

$$\begin{array}{lrcl} \max_{w,h^{tp}} & \sum_{i=1}^{m} E[f^i]w_i - \lambda(h^{tp})^T Q h^{tp}, & & \\ \text{s.t.} & h^{tp} - \sum_{i=1}^{m} w_i h^i & = & 0 \end{array} \tag{8}$$

for the target portfolio as it better highlights the relationship between the optimization problems in the three stages of the consistent investment process. Note that one can also force the target portfolio to satisfy some of the additional constraints in $\mathcal{C}$. If this is the case, then one can also add the constraints $h^{tp} \in \mathcal{C}_{tp}$, where $\mathcal{C}_{tp}$ is subset of the constraints in the final portfolio construction problem, to the MVO optimization problem (8).

3. If the FMPs h^i used to generate the target portfolio are all pure, then the FMP returns represent the underlying alpha factor returns if the alpha factors are also in the risk model. These factor returns are usually constructed from a cross-sectional regression model. In this case, one can set $\Theta = \Omega_A$ in (7), where Ω_A is the factor covariance matrix constructed from the factor returns. The corresponding problem (8) has Q constructed from the factor portion $X\Omega X^T$ of the custom risk model Q.

We must emphasize that for the target portfolio optimization problem (8) we only have m unknowns, i.e., the number of FMPs that need to be combined to form the target portfolio. Usually, $m << n$, where n is the number of assets in the portfolio. As a result, *asset-level* constraints in the target portfolio problem may be too restrictive.

3.3 Solving the portfolio construction problem

The target portfolio generated in the second phase of the consistent process may violate some of the constraints in the final portfolio construction problem which are included in $\mathcal{C}$. The third phase of the consistent investment process constructs a final portfolio that satisfies all the constraints in $\mathcal{C}$ and is as close to the target portfolio as possible. This problem is solved with an MVO model that uses as vector of expected returns the implied alpha of the target portfolio. In other words, the vector of expected returns is chosen as

$$\tilde{\alpha}^{tp} = Q h^{tp} \tag{9}$$

where Q is the risk model in 1. The portfolio optimization in the third phase can be written as

$$\max_{h \in \mathcal{C}} \quad (\tilde{\alpha}^{tp})^T h - \frac{\lambda}{2} h^T Q h \tag{10}$$

where $\lambda > 0$ is an appropriate risk aversion parameter and $\mathcal{C}$ contains all the constraints in the final portfolio optimization problem.

Let us briefly motivate the choice of the implied alpha $\tilde{\alpha}^{tp}$ as the alpha signal in the portfolio construction problem. In the unconstrained setting, i.e., when the constraint set $\mathcal{C}$ is empty, the optimal holdings are given by

$$h^* = \frac{1}{\lambda} h^{tp}. \quad (11)$$

In other words, in the unconstrained setting, the final portfolio h^* is a multiple of the target portfolio h^{tp}. Similarly, one can show that choosing the alpha as the implied alpha of the target portfolio in the constrained setting actually gives a final portfolio that is close to a multiple of the target portfolio h^{tp}.

4 Illustrative example

Consider a portfolio manager who wants to combine three alpha signals in a consistent investment process. The three alpha signals are *Value* (Sales to Price), *Momentum* (assets cumulative return over the last 250 trading days), and *Quality* (Return on Equity). The asset universe and the benchmark is restricted to be the FTSE All-World. The objective is to maximize the expected return subject to

1. Long-Only and Fully Invested.
2. Round-Trip (two-way) Turnover restricted to be at most 15% per month.
3. Active Predicted Beta Bounds of ±2%.
4. Active Industry and Country Bounds of ±2%.
5. Maximum Predicted Active Risk of 3%.
6. Axioma Style Exposure Bounds of ±10% on Liquidity, Leverage, Size, Exchange-Rate Sensitivity, and Volatility.

Our backtest period is from December 2000 to August 2012 where the portfolio is rebalanced at the end of each month. We construct a custom risk model using Axioma's Risk Model Machine (RMM) that also includes the Value, Momentum, and Quality alpha signals.

We now discuss the three stages of the consistent investment process as applied to this example.

1. The Value, Momentum, and Quality FMPs are constructed with $W = M^{-1}$ where M is the diagonal matrix whose entries are the asset market-caps.
   (a) The Value FMP is neutral to countries.
   (b) The Momentum FMP is neutral to industries.
   (c) The Quality FMP is neutral to industries and size.

We will motivate our choices for these FMPs below.

2. The optimization problem that generates the target portfolio is

$$\begin{aligned} \max_{w \geq 0, h^{tp} \in \mathcal{C}_{tp}} \quad & \sum_{i=1}^{3} E[f^i] w_i + (h^{tp})^T Q h^{tp}, \\ \sqrt{(h^{tp})^T Q h^{tp}} &\leq 3\%, \\ h^{tp} - \sum_{i=1}^{3} w_i h^i &= 0, \end{aligned} \tag{12}$$

where we additionally impose that h^{tp} belong to the set $\mathcal{C}_{tp}$. We will highlight some of our choices of this set later in this Section. Note that we impose a risk constraint on the target portfolio that is also one of the constraints in our final strategy. Since we expect each of the alpha signals to add value, we also impose a non-negativity restriction of the FMP weights. The $E[f^i]$ is taken to be the long-term average return for the three FMPs and is constant for the backtest.

3. The expected return for the final portfolio problem is the implied alpha of the target portfolio.

Let us highlight how we arrived at our choice for country neutrality constraints for the Value FMP. For each time period, we first constructed a simple dollar-neutral Value FMP with $W = M^{-1}$ with no factor neutrality constraints. We then ran Axioma's factor Performance Attribution Tool to analyze the resulting FMP. The results are presented in Tables 1 and 2. Table 1 indicates that the FMP has a large negative return betting on countries. Table

Source of Return	Contribution	IR
FMP	2.49%	0.51
Factor Contribution	3.61%	0.64
Axioma Style	2.21%	1.27
Custom Style	2.57%	0.72
Country	-1.72%	-0.48
Industry	0.70%	0.37
Currency	-0.15%	-0.08
Market	0.00%	-0.31
Specific Return	-1.12%	-0.50

Table 1: Factor return contribution in dollar-neutral Value FMP

2 further indicates that the FMP is taking significant positive and negative exposures on countries. For example, the FMP has a large negative exposure to Japan (-14.15%) and a large positive exposure (8.00%) to UK. So, we made the FMP neutral to all the countries to force it to bet on stocks within countries rather than on individual countries. The resulting

Source of Return Notable Factors	Return Contribution	Avg Exposure
Volatility	1.51%	-8.28%
Value	2.61%	100%
China	-0.08%	-2.34%
France	-0.24%	5.59%
Germany	-0.20%	5.81%
Japan	-0.28%	-14.15%
Korea	-0.07%	-2.16%
UK	-0.33%	8.00%
USA	0.11%	8.10%
Banks	0.05%	-6.74%
Food & Staples	-0.12%	4.78%
Oil, Gas & Consumable Fuels	0.54%	6.47%

Table 2: Factors with the largest exposures in dollar-neutral Value FMP

FMP is the country-neutral value FMP and it is the value FMP that we will use in the rest of the section. Table 3 compares the dollar-neutral and the country-neutral value FMPs. Note that country-neutral FMPs are also dollar-neutral. Enforcing country-neutrality removes the

	Country-Neutral Value FMP			Dollar-Neutral Value FMP		
Return	Contribution	Risk	IR	Contribution	Risk	IR
FMP	3.87%	4.90%	0.79	2.49%	4.90%	0.51
Factor	4.92%	5.21%	0.94	3.61%	5.67%	0.64
Axioma Style	1.51%	1.84%	0.82	2.21%	1.74%	1.27
Custom Style	2.57%	3.61%	0.71	2.67%	3.59%	0.72
Country	0.00%	0.00%	-	-1.72%	3.59%	-0.48
Industry	0.83%	2.26%	0.37	0.70%	1.91%	0.37
Currency	0.00%	0.00%	-	-0.15%	1.84%	-0.08
Market	0.00%	0.00%	-	0.00%	0.00%	-
Specific Return	-1.04%	2.06%	-0.51	-1.12%	2.26%	-0.5

Table 3: Comparing dollar-neutral and country-neutral value FMPs

negative contribution to the return from the country factors. The portfolio IR improves from 0.51 to 0.79. We experimented with different neutrality settings for the Value, Momentum, and Quality FMPs. The IRs for these different settings are summarized in Table 4. The Value FMPs perform better with the market-cap weighting. We chose the FMP that was neutral to the country factors; this had the best IR in Table 4. For Momentum, we first constructed a dollar-neutral FMP with market-cap weighting. A factor performance attribution analysis

FMP	Weighting (W)	IR			
		None	IndNeutral	CountryNeutral	IndAndCountryNeutral
Value	M^{-1}	0.50	0.20	0.78	0.60
Value	I	0.12	0.05	0.70	0.56
Momentum	M^{-1}	0.30	0.24	0.24	0.12
Momentum	I	0.36	0.26	0.36	0.22
Quality	M^{-1}	0.56	0.53	0.44	0.28
Quality	I	0.77	0.74	0.52	0.48

Table 4: IRs for different Value, Momentum, and Quality FMPs

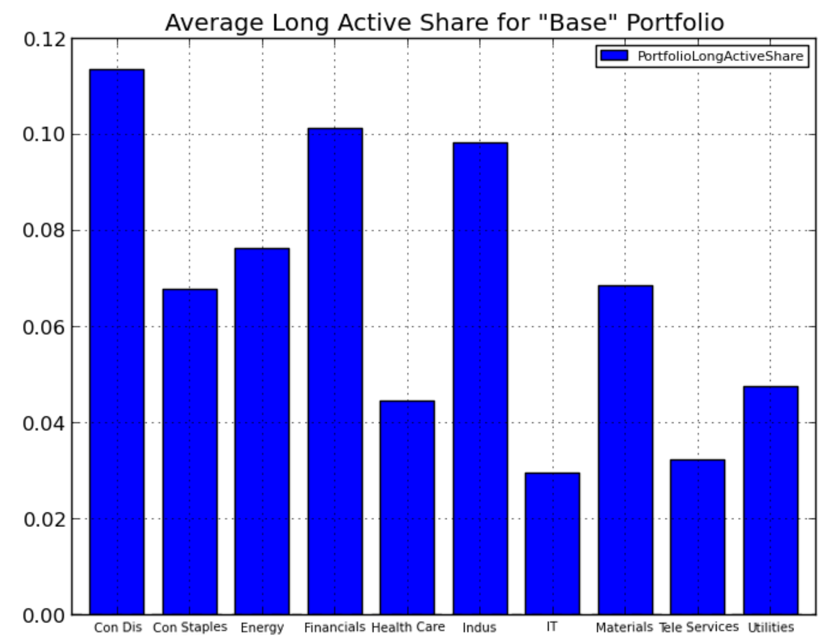
indicated that this FMP did not take any sizeable exposures to the industry and country factors. We settled on the Momentum FMP with market-cap weighting that is neutral to the industries since available literature (see Asness (1997)) indicates that Momentum performs better with industry neutrality. Similarly, for Quality, we first constructed a dollar-neutral FMP with market-cap weighting. A factor PA analysis indicated that this FMP had a 10.78% exposure to the size factor that translated into a negative return of −0.48%. Moreover, the Quality FMP did not take any sizeable exposures to the industry and country factors either. We finally chose the Quality FMP with market cap weighting that is both industry and size neutral.

We ran our *Base* backtest with these FMPs and following the consistent investment process that we outlined earlier. The results are summarized in Table 5. Figure 1 compares

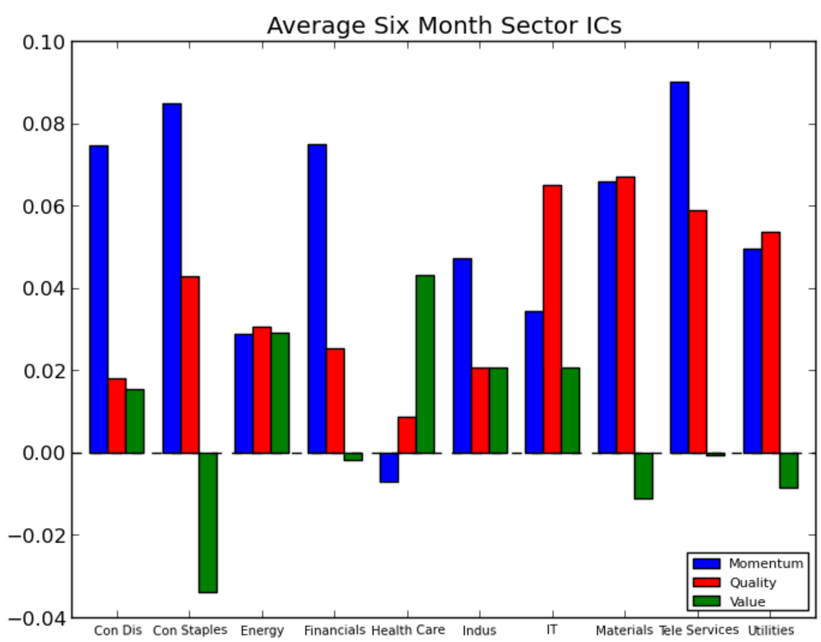
	Base
Annualized Active Return	2.27%
Avg Annualized Active Risk	2.83%
Avg Turnover	14.89%
Information Ratio	0.801

Table 5: Base backtest summary

the average active long holdings of the final portfolio to the average six month horizon ICs for the three alpha signals in each sector of the economy. Note that the quality of the three alpha signals varies substantially in each sector. Also, note that average six month sector ICs for the Value signal are very poor and so we focus on the strength of the Momentum and the Quality signals in each sector. These signals both appear to be very strong in the *Telecommunication Services* sector but the *Base* portfolio seems to take relatively small bets in this sector. In general, we see that the sector bets of the *Base* portfolio are not proportional to the sector ICs of the Momentum and Quality signals. This is not surprising since the active beta, industry, and country bound constraints require the portfolio to take very small exposures to these factors. As a result, the portfolio bets are concentrated in certain sectors or countries. So, we add additional constraints to the final strategy that limits the long holding in each sector to be at most 5% of the portfolio size. We refer to



(a) Active Share in Each Sector



(b) Average Six Month Sector ICs

Figure 1: Comparing the average long active holdings to the average six month horizons ICs for alpha signals in each sector

these new constraints as the *active share* constraints. We ran a second backtest called *Base + CAS* (CAS = Constrained Active Share) that is identical to the base strategy with the only difference being the active share constraints in the final strategy. The results of this backtest are given in Table 6. Note that the IR of the portfolio has improved from 0.8 to 0.85. This is the typical process employed by most portfolio managers where additional constraints are added in the final portfolio problem to overcome some shortcomings. The addition of the constraints in the final portfolio, however, reduces the TC. Our objective in the consistent investment process is to target a high TC in order to improve the transparency of the portfolio construction process. With this in mind, we construct our FMPs and the target portfolio to reflect these active share constraints.

	Base + CAS
Annualized Active Return	2.36%
Avg Annualized Active Risk	2.78%
Avg Turnover	14.89%
Information Ratio	0.85

Table 6: Base + CAS backtest summary

We constructed a second set of momentum and quality FMPs where the active share in each sector is restricted to 5% of the portfolio size. Note that the quality of the value signal is quite poor in most of the sectors and so we persist with the same value FMP. We generated a new target portfolio and implied alpha by combining these modified momentum and quality FMPs and the original value FMP. We ran a third backtest called *BaseCFMP* (CFMP = Constrained FMP), where the active share constraints are used in the FMP generation (phase 1) of the consistent investment process rather than the final strategy. The results are summarized in Table 7. Note that IR has further increased to 1.01. Next we ran another backtest called *BaseCTP* (CTP = Constrained Target Portfolio), where the active share constraints are used in the target portfolio construction (phase 2) of the consistent investment process. The results are summarized in Table 8. This further increases the IR to 1.10. Note that the only difference between Base + CAS, BaseCFMP, and BaseCTP is that the active share constraints are imposed at different stages of the consistent investment process. Each approach incrementally improved the IR of the portfolio.

	BaseCFMP
Annualized Active Return	2.95%
Avg Annualized Active Risk	2.94%
Avg Turnover	14.89%
Information Ratio	1.01

Table 7: BaseCFMP backtest summary

We then ran three more backtests *BaseCTP + CAS*, where the active share constraints are imposed both in the target and final portfolios; *Base CFMPTP* (CFMPTP = Constrained

	BaseCTP
Annualized Active Return	3.29%
Avg Annualized Active Risk	2.99%
Avg Turnover	14.89%
Information Ratio	1.10

Table 8: BaseCFMP backtest summary

FMP and Target Portfolio), where the active share constraints are imposed in the momentum and quality FMPs and the final portfolio; and finally *Base CFMPTP + CAS*, where the active share constraints are imposed in all the stages of the consistent investment process, i.e., in the generation of the momentum and quality FMPs, the target portfolio, as well as the final portfolio. The results are summarized in Table 9. We note that adding the

	BaseCTP + CAS	BaseCFMPTP	BaseCFMPTP + CAS
Annualized Active Return	3.29%	3.37%	3.32%
Avg Annualized Active Risk	2.84%	3.08%	2.94%
Avg Turnover	14.89%	14.89%	14.89%
Information Ratio	1.15	1.09	1.13

Table 9: BaseCFTP + CAS, Base CFMPTP, and BaseCFMPTP + CAS summaries

active share constraints to the final portfolio ensures that the final portfolio also satisfies these constraints. Moreover, the addition of these constraints also improves the IR for the BaseCFTP and BaseCFMPTP backtests.

We conclude this section with Figure 2 that compares the differences in the TCs between the (BaseCTP + CAS) and BaseCTP backtests with the corresponding differences between the (Base + CAS) and Base backtests. Note that there is less deterioration in the TC when one adds the active share constraints in the final portfolio problem in the BaseCTP backtest. This highlights our comment that it is useful to consider some of the final portfolio constraints in the construction of the implied alpha signal. Ensuring that the target portfolio also satisfies these constraints helps limit the deterioration of the TC in the realistic portfolio construction problem.

5 Conclusions

We presented a use case for the consistent investment approach presented in Stubbs (2013). The process breaks the original portfolio construction into three portfolio optimization problems that

1. Transform each alpha signal in the alpha factor into a factor mimicking portfolio (FMP) that represents the signal. Some of the signal specific constraints in the final portfolio problem can be added to the optimization problem that generates the FMP.

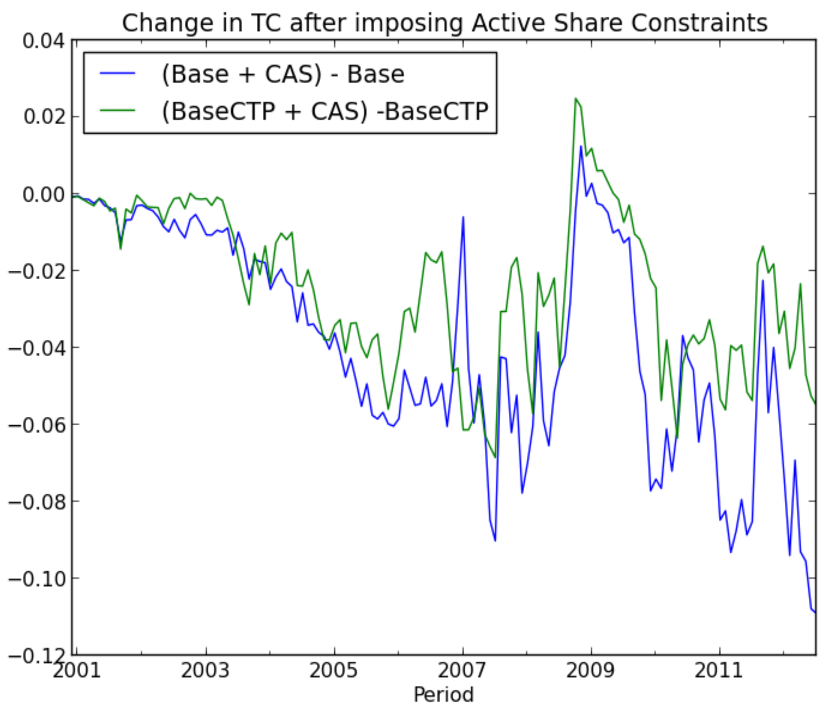


Figure 2: Change in the TCs after the addition of the Active Share constraints

2. Combine the factor mimicking portfolios into a target portfolio by solving an MVO optimization problem. Some of the constraints in the final portfolio problem can be added to the optimization problem that generates the target portfolio.

3. Solve the actual portfolio construction problem with two changes: The first is a new alpha signal that is constructed as the implied alpha of the target portfolio and the use of a custom risk model. The second is that this portfolio problem only has the relevant implementation constraints. The final stage of the consistent process generates a portfolio that satisfies the relevant implementation constraints, and is also close to the target portfolio that is constructed in Step 2.

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A Technical Appendix

Proposition 1 *The FMP (5) with only the industry neutrality constraints carries out the equal and market cap weighted industry purifications when $W = I$ and $W = M^{-1}$ where M is the diagonal matrix whose entries are the asset market-caps, respectively.*

Proof: Consider the FMP problem

$$\begin{array}{ll} \min\limits_h & \frac{1}{2}h^T W h \\ \text{s.t.} & \begin{array}{rcl} X_I^T h & = & 0, \\ \alpha^T h & = & 1 \end{array} \end{array} \tag{13}$$

that is neutral to the industry factors and where α is our alpha signal. In this case, the optimal portfolio h^* has the following expression

$$h^* = \theta W^{-1/2}\left(I - W^{-1/2}X_I\left(X_I^T W^{-1} X_I\right)^{-1} X_I^T W^{-1/2}\right) W^{-1/2}\alpha \tag{14}$$

where

$$\theta = \alpha^T W^{-1/2}\left(I - W^{-1/2}X_I\left(X_I^T W^{-1} X_I\right)^{-1} X_I^T W^{-1/2}\right) W^{-1/2}\alpha \tag{15}$$

is a positive constant. The equal and the market-cap weighted industry purifications update the α as

$$\bar{\alpha} = (I - X_I(X_I^T W^{-1} X_I)^{-1} X_I^T W^{-1})\alpha, \tag{16}$$

where $W = I$ and $W = M^{-1}$, respectively. Consider the unconstrained MVO problem

$$\min_h \quad \bar{\alpha}^T h - \frac{1}{2\theta} h^T W h \tag{17}$$

with the industry purified alpha, where θ is given by (15). The solution to this problem is given by

$$\begin{aligned} h^{MVO} &= \theta W^{-1}\bar{\alpha} \\ &= \theta W^{-1}\left(I - X_I\left(X_I^T W^{-1} X_I\right)^{-1} X_I^T W^{-1}\right)\alpha \\ &= \theta W^{-1/2}\left(I - W^{-1/2}X_I\left(X_I^T W^{-1} X_I\right)^{-1} X_I^T W^{-1/2}\right) W^{-1/2}\alpha. \end{aligned} \tag{18}$$

Note that h^* in (14) is identical to the h^{MVO} in (18). In other words, the optimization problem (13) is implicitly neutralizing the alpha signal over the industries using the weighted projection in (16).

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